Equivalence of Left Linear, Right Linear and Regular Languages

Dr. Neminath Hubballi

Indian Institute of Technology Indore

January 29, 2018

Overview

1. Reverse of a Regular Language
2. Left Linear Grammar to Right Linear Grammar

Converting L into L^R

Theorem

If L is a regular language then L^R is also regular

Proof:

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Proof: Let L be a regular language
$\Rightarrow \exists$ an nfa such that $L = L(M)$

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Proof: Let L be a regular language
$\Rightarrow \exists$ an nfa such that $L = L(M)$
M=$(Q, \Sigma, \delta, q_0, F)$ (M has a single final state)

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M=$(Q, \Sigma, \delta, q_0, F)$ (M has a single final state)
Construct a new machie $M^{'}$ =$(Q^{'}, \Sigma^{'}, \delta^{'}, q_f, F^{'})$ (with $q_0 \in Q$)

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$Q^{'} = Q$

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$Q^{'} = Q$
$\Sigma^{'} = \Sigma$

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$Q^{'} = Q$
$\Sigma^{'} = \Sigma$
$F^{'} = \{q_0\}$

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$Q^{'} = Q$
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Add transitions as follows

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$Q^{'} = Q$
$\Sigma^{'} = \Sigma$
$F^{'} = \{q_0\}$
Add transitions as follows
For every transition $\delta(q_i, t) = q_j$ in M (where $t \in \Sigma \bigcup \lambda$)

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Construct a new machie $M^{'}$ =$(Q^{'}, \Sigma^{'}, \delta^{'}, q_f, F^{'})$ (with $q_0 \in Q$)
$Q^{'} = Q$
$\Sigma^{'} = \Sigma$
$F^{'} = \{q_0\}$
Add transitions as follows
For every transition $\delta(q_i, t) = q_j$ in M (where $t \in \Sigma \bigcup \lambda$)
Add a transition $\delta^{'}(q_j, t) = q_i$

Converting L into L^R

Correctness Claim: Let $w = t_1 t_2, \cdots, t_n \in L(M)$

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$\Rightarrow \delta^*(q_0, w) = q_f$
$\Rightarrow \delta^*(q_0, t_1 t_2, \cdots, t_n) = q_f$

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Correctness Claim: Let $w = t_1 t_2, \cdots, t_n \in L(M)$
$\Rightarrow \delta^*(q_0, w) = q_f$
$\Rightarrow \delta^*(q_0, t_1 t_2, \cdots, t_n) = q_f$
$\Rightarrow \delta^*(\delta(q_0, t_1), t_2, \cdots, t_n) = q_f$

Converting L into L^R

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$\Rightarrow \delta^*(q_0, t_1 t_2, \cdots, t_n) = q_f$
$\Rightarrow \delta^*(\delta(q_0, t_1), t_2, \cdots, t_n) = q_f$
$\Rightarrow \delta^*(q_a, t_2, \cdots, t_n) = q_f$

Correctness Claim: Let $w = t_1 t_2, \cdots, t_n \in L(M)$
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$\Rightarrow \delta^*(q_0, t_1 t_2, \cdots, t_n) = q_f$
$\Rightarrow \delta^*(\delta(q_0, t_1), t_2, \cdots, t_n) = q_f$
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$\Rightarrow \delta^*(\delta(q_a, t_2), t_3, \cdots, t_n) = q_f$

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Correctness Claim: Let $w = t_1 t_2, \cdots, t_n \in L(M)$
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$\Rightarrow \delta^*(\delta(q_0, t_1), t_2, \cdots, t_n) = q_f$
$\Rightarrow \delta^*(q_a, t_2, \cdots, t_n) = q_f$
$\Rightarrow \delta^*(\delta(q_a, t_2), t_3, \cdots, t_n) = q_f$
$\ldots \Rightarrow \delta(q_x, t_n) = q_f$

Converting L into L^R

Correctness Claim: Let $w = t_1 t_2, \cdots, t_n \in L(M)$

$\Rightarrow \delta^*(q_0, w) = q_f$

$\Rightarrow \delta^*(q_0, t_1 t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(\delta(q_0, t_1), t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(q_a, t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(\delta(q_a, t_2), t_3, \cdots, t_n) = q_f$

$. \ . \ . \Rightarrow \delta(q_x, t_n) = q_f$

$w^R = t_n, t_{n-1}, \cdots, t_1$

Converting L into L^R

Correctness Claim: Let $w = t_1 t_2, \cdots, t_n \in L(M)$

$\Rightarrow \delta^*(q_0, w) = q_f$

$\Rightarrow \delta^*(q_0, t_1 t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(\delta(q_0, t_1), t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(q_a, t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(\delta(q_a, t_2), t_3, \cdots, t_n) = q_f$

$. \;.\; . \Rightarrow \delta(q_x, t_n) = q_f$

$w^R = t_n, t_{n-1}, \cdots, t_1$

To show

$\Rightarrow \delta'^*(q_f, w^R) = q_0$

Converting L into L^R

Correctness Claim: Let $w = t_1 t_2, \cdots, t_n \in L(M)$
$\Rightarrow \delta^*(q_0, w) = q_f$
$\Rightarrow \delta^*(q_0, t_1 t_2, \cdots, t_n) = q_f$
$\Rightarrow \delta^*(\delta(q_0, t_1), t_2, \cdots, t_n) = q_f$
$\Rightarrow \delta^*(q_a, t_2, \cdots, t_n) = q_f$
$\Rightarrow \delta^*(\delta(q_a, t_2), t_3, \cdots, t_n) = q_f$
$. \ . \ . \Rightarrow \delta(q_x, t_n) = q_f$
$w^R = t_n, t_{n-1}, \cdots, t_1$
To show
$\Rightarrow \delta'^*(q_f, w^R) = q_0$
$\Rightarrow \delta'^*(q_f, t_n t_{n-1}, \cdots, t_1) = q_0$

Correctness Claim: Let $w = t_1 t_2, \cdots, t_n \in L(M)$

$\Rightarrow \delta^*(q_0, w) = q_f$

$\Rightarrow \delta^*(q_0, t_1 t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(\delta(q_0, t_1), t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(q_a, t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(\delta(q_a, t_2), t_3, \cdots, t_n) = q_f$

$. \ . \ . \Rightarrow \delta(q_x, t_n) = q_f$

$w^R = t_n, t_{n-1}, \cdots, t_1$

To show

$\Rightarrow \delta^{'*}(q_f, w^R) = q_0$

$\Rightarrow \delta^{'*}(q_f, t_n t_{n-1}, \cdots, t_1) = q_0$

$\Rightarrow \delta^{'*}(\delta^{'}(q_f, t_n), t_{n-1}, \cdots, t_1)$

Converting L into L^R

Correctness Claim: Let $w = t_1 t_2, \cdots, t_n \in L(M)$

$\Rightarrow \delta^*(q_0, w) = q_f$

$\Rightarrow \delta^*(q_0, t_1 t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(\delta(q_0, t_1), t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(q_a, t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(\delta(q_a, t_2), t_3, \cdots, t_n) = q_f$

$. \ . \ . \Rightarrow \delta(q_x, t_n) = q_f$

$w^R = t_n, t_{n-1}, \cdots, t_1$

To show

$\Rightarrow \delta'^*(q_f, w^R) = q_0$

$\Rightarrow \delta'^*(q_f, t_n t_{n-1}, \cdots, t_1) = q_0$

$\Rightarrow \delta'^*(\delta'(q_f, t_n), t_{n-1}, \cdots, t_1)$

$\Rightarrow \delta'^*(q_x, t_{n-1}, \cdots, t_1)$ $(\because \delta(q_x, t_n) = q_f \in M \Rightarrow \delta'(q_f, t_n) = q_x \in M')$

Converting L into L^R

Correctness Claim: Let $w = t_1 t_2, \cdots, t_n \in L(M)$

$\Rightarrow \delta^*(q_0, w) = q_f$

$\Rightarrow \delta^*(q_0, t_1 t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(\delta(q_0, t_1), t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(q_a, t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(\delta(q_a, t_2), t_3, \cdots, t_n) = q_f$

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$w^R = t_n, t_{n-1}, \cdots, t_1$

To show

$\Rightarrow \delta'^*(q_f, w^R) = q_0$

$\Rightarrow \delta'^*(q_f, t_n t_{n-1}, \cdots, t_1) = q_0$

$\Rightarrow \delta'^*(\delta'(q_f, t_n), t_{n-1}, \cdots, t_1)$

$\Rightarrow \delta'^*(q_x, t_{n-1}, \cdots, t_1)$ $(\because \delta(q_x, t_n) = q_f \in M \Rightarrow \delta'(q_f, t_n) = q_x \in M')$

$.... \Rightarrow \delta(q_a, t_1) = q_0 \in F'$

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Correctness Claim: Let $w = t_1 t_2, \cdots, t_n \in L(M)$

$\Rightarrow \delta^*(q_0, w) = q_f$

$\Rightarrow \delta^*(q_0, t_1 t_2, \cdots, t_n) = q_f$

$\Rightarrow \delta^*(\delta(q_0, t_1), t_2, \cdots, t_n) = q_f$

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$w^R = t_n, t_{n-1}, \cdots, t_1$

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$\Rightarrow \delta'^*(q_f, w^R) = q_0$

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$\Rightarrow \delta'^*(q_x, t_{n-1}, \cdots, t_1)$ $(\because \delta(q_x, t_n) = q_f \in M \Rightarrow \delta'(q_f, t_n) = q_x \in M')$

$.... \Rightarrow \delta(q_a, t_1) = q_0 \in F'$

Converting Left Linear Grammar to Right Linear Grammar

Theorem

If L is a regular language, then there exist an equivalent Left Linear Grammar $G^{'}$ such that $L = L(G^{'})$

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Proof: Let L be a regular language

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If L is a regular language, then there exist an equivalent Left Linear Grammar $G^{'}$ such that $L = L(G^{'})$

Proof: Let L be a regular language
$\Rightarrow \exists$ and nfa M=$(Q, \Sigma, \delta, q_0, F)$ such that L=L(M)

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If L is a regular language, then there exist an equivalent Left Linear Grammar $G^{'}$ such that $L = L(G^{'})$

Proof: Let L be a regular language
$\Rightarrow \exists$ and nfa M=$(Q, \Sigma, \delta, q_0, F)$ such that L=L(M)
Construct a new machine $M^{'}$ for L^R

Converting Left Linear Grammar to Right Linear Grammar

Theorem

If L is a regular language, then there exist an equivalent Left Linear Grammar G' such that $L = L(G')$

Proof: Let L be a regular language
$\Rightarrow \exists$ and nfa M=$(Q, \Sigma, \delta, q_0, F)$ such that L=L(M)
Construct a new machine M' for L^R
Generate a Right Linear Grammar G for L^R

Converting Left Linear Grammar to Right Linear Grammar

Theorem

If L is a regular language, then there exist an equivalent Left Linear Grammar $G^{'}$ such that $L = L(G^{'})$

Proof: Let L be a regular language
$\Rightarrow \exists$ and nfa M=$(Q, \Sigma, \delta, q_0, F)$ such that L=L(M)
Construct a new machine $M^{'}$ for L^R
Generate a Right Linear Grammar G for L^R
Generate a Left Linear Grammar $G^{'}$ by rewriting production rules

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If L is a regular language, then there exist an equivalent Left Linear Grammar $G^{'}$ such that $L = L(G^{'})$

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Construct a new machine $M^{'}$ for L^R
Generate a Right Linear Grammar G for L^R
Generate a Left Linear Grammar $G^{'}$ by rewriting production rules

1. For every production $X- > xY$ (where $x \in T^*$)

Converting Left Linear Grammar to Right Linear Grammar

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Construct a new machine $M^{'}$ for L^R
Generate a Right Linear Grammar G for L^R
Generate a Left Linear Grammar $G^{'}$ by rewriting production rules

1. For every production $X- > xY$ (where $x \in T^*$)
   Add a production in $G^{'}$ $X- > Yx^R$

Converting Left Linear Grammar to Right Linear Grammar

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If L is a regular language, then there exist an equivalent Left Linear Grammar $G^{'}$ such that $L = L(G^{'})$

Proof: Let L be a regular language
$\Rightarrow \exists$ and nfa M=$(Q, \Sigma, \delta, q_0, F)$ such that L=L(M)
Construct a new machine $M^{'}$ for L^R
Generate a Right Linear Grammar G for L^R
Generate a Left Linear Grammar $G^{'}$ by rewriting production rules

1. For every production $X -> xY$ (where $x \in T^*$)
   Add a production in $G^{'}$ $X -> Yx^R$
2. For every production $X -> x$ (where $x \in T^*$)

Theorem

If L is a regular language, then there exist an equivalent Left Linear Grammar G' such that $L = L(G')$

Proof: Let L be a regular language
$\Rightarrow \exists$ and nfa M=$(Q, \Sigma, \delta, q_0, F)$ such that L=L(M)
Construct a new machine M' for L^R
Generate a Right Linear Grammar G for L^R
Generate a Left Linear Grammar G' by rewriting production rules

1. For every production $X -> xY$ (where $x \in T^*$)
   Add a production in G' $X -> Yx^R$
2. For every production $X -> x$ (where $x \in T^*$)
   Add a production in G' $X -> x^R$

Converting Left Linear Grammar to Right Linear Grammar

Theorem

If L is a regular language, then there exist an equivalent Left Linear Grammar $G^{'}$ such that $L = L(G^{'})$

Proof: Let L be a regular language
$\Rightarrow \exists$ and nfa M=$(Q, \Sigma, \delta, q_0, F)$ such that L=L(M)
Construct a new machine $M^{'}$ for L^R
Generate a Right Linear Grammar G for L^R
Generate a Left Linear Grammar $G^{'}$ by rewriting production rules

1. For every production $X- > xY$ (where $x \in T^*$)
   Add a production in $G^{'}$ $X- > Yx^R$
2. For every production $X- > x$ (where $x \in T^*$)
   Add a production in $G^{'}$ $X- > x^R$

Correctness: L=L(M)=$L(M^{'})^R$=$L(G)^R$=$L(G^{'})$

Theorem

If L is a language defined by a Left Linear Grammar G (i.e., L=L(G)), then L is a regular language

Proof: Take it as exercise !

Converting Right Linear Grammar to Left Linear Grammar

Theorem

If G is a Right Linear Grammar, then there is a Left Linear Grammar G'' such that $L(G) = L(G'')$

Converting Right Linear Grammar to Left Linear Grammar

Theorem

If G is a Right Linear Grammar, then there is a Left Linear Grammar $G^{''}$ such that $L(G) = L(G^{''})$

Proof: Conversion outline (Not a rigorous proof!)

Converting Right Linear Grammar to Left Linear Grammar

Theorem

If G is a Right Linear Grammar, then there is a Left Linear Grammar G'' *such that* $L(G) = L(G'')$

Proof: Conversion outline (Not a rigorous proof!)

1. From G, construct M

Converting Right Linear Grammar to Left Linear Grammar

Theorem

If G is a Right Linear Grammar, then there is a Left Linear Grammar $G^{''}$ *such that* $L(G) = L(G^{''})$

Proof: Conversion outline (Not a rigorous proof!)

1. From G, construct M
2. From M construct $M^{'}$ for L^R

Converting Right Linear Grammar to Left Linear Grammar

Theorem

If G is a Right Linear Grammar, then there is a Left Linear Grammar G'' such that $L(G) = L(G'')$

Proof: Conversion outline (Not a rigorous proof!)

1. From G, construct M
2. From M construct M' for L^R
3. Generate G' a Right Linear Grammar for L^R

Converting Right Linear Grammar to Left Linear Grammar

Theorem

If G is a Right Linear Grammar, then there is a Left Linear Grammar G'' such that $L(G) = L(G'')$

Proof: Conversion outline (Not a rigorous proof!)

1. From G, construct M
2. From M construct M' for L^R
3. Generate G' a Right Linear Grammar for L^R
4. Generate G'' a Left Linear Grammar for L^{R^R}

Converting Right Linear Grammar to Left Linear Grammar

Theorem

If G is a Left Linear Grammar, then there is a Right Linear Grammar $G^{''}$ such that $L(G) = L(G^{''})$

Converting Right Linear Grammar to Left Linear Grammar

Theorem

If G is a Left Linear Grammar, then there is a Right Linear Grammar G'' *such that* $L(G) = L(G'')$

Proof: Conversion outline (Not a rigorous proof!)

Converting Right Linear Grammar to Left Linear Grammar

Theorem

If G is a Left Linear Grammar, then there is a Right Linear Grammar G'' *such that* $L(G) = L(G'')$

Proof: Conversion outline (Not a rigorous proof!)

1. From G, generate G' for L^R

Theorem

If G is a Left Linear Grammar, then there is a Right Linear Grammar G'' *such that* $L(G) = L(G'')$

Proof: Conversion outline (Not a rigorous proof!)

1. From G, generate G' for L^R
2. From G' construct M for L^R

Converting Right Linear Grammar to Left Linear Grammar

Theorem

If G is a Left Linear Grammar, then there is a Right Linear Grammar $G^{''}$ such that $L(G) = L(G^{''})$

Proof: Conversion outline (Not a rigorous proof!)

1. From G, generate $G^{'}$ for L^R
2. From $G^{'}$ construct M for L^R
3. Construct $M^{'}$ from M for L^{R^R}

Converting Right Linear Grammar to Left Linear Grammar

Theorem

If G is a Left Linear Grammar, then there is a Right Linear Grammar G'' such that $L(G) = L(G'')$

Proof: Conversion outline (Not a rigorous proof!)

1. From G, generate G' for L^R
2. From G' construct M for L^R
3. Construct M' from M for L^{R^R}
4. Generate G'' a Right Linear Grammar for L^{R^R}

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If G is a Left Linear Grammar, then there is a Right Linear Grammar G'' such that $L(G) = L(G'')$

Proof: Conversion outline (Not a rigorous proof!)

1. From G, generate G' for L^R
2. From G' construct M for L^R
3. Construct M' from M for L^{R^R}
4. Generate G'' a Right Linear Grammar for L^{R^R}