Game-theoretic Green Electric Vehicle Energy Networks Management in Smart Grid

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***Abstract*—In this paper, the problem of green electric vehicle energy networks management is studied using a non-cooperative Stackelberg game theoretic model. The micro-grid acts as the leader, and needs to decide the price per unit energy based on the amount of energy requested by the plug-in hybrid electric vehicles (PHEVs) for charging and the amount of energy to be requested to each PHEV for discharging, respectively. On the other hand, the PHEVs act as the followers, and need to decide the amount energy to be consumed and the price per unit energy for charging and discharging, respectively, in order to reduce CO_2 emission, i.e., using green energy. Using the proposed electric vehicle energy networks management (EVENT) scheme, there exists Nash equilibrium solution for charging and discharging, and the satisfaction factors are high using proposed EVENT scheme.**

***Keywords*—Electric Vehicle, Energy Networks, Energy Management, Charging, Discharging, Non-cooperative Game Theory, Stackelberg Game**

I. Introduction

To achieve high reliability in power systems, traditional electrical grids need to be designed as modernized electrical systems, termed as *smart grids*. Unlike the existing power systems, in which electricity is distributed unidirectionally to the customers by the main grid having a centralized system, in a smart grid with duplex communication infrastructure, the large scale traditional electrical grid is divided into the micro-grids having bi-directional electricity exchange facility and green energy generation capacity with low CO_2 emission. It is desirable to allow each micro-grid to service a small geographical area based on their demands in a distributed manner, so as to relax the load on the main grid.

The micro-grid generates energy using renewable energy resources, i.e., green energy, such as wind power, solar energy, and hydro power. So, the generated amount of energy is not fixed. Therefore, the micro-grids may have excess amount of generated energy in any time-slot, whereas the micro-grid may have deficiency of generated energy in another time slot. In both the situations, energy is not properly utilized. Therefore, we need to have a green energy management scheme. In smart grid, plug-in hybrid electric vehicles (PHEVs) help to manage the energy network through charging and discharging. Therefore, we propose a *game-theoretic green electric vehicle energy networks management (EVENT)* scheme. Using our proposed scheme, the PHEVs can charge themselves, when the micro-grid is having excess amount of energy, and the PHEVs supply energy to the micro-grid and make profit, when the micro-grid has a deficiency of energy. In the last few years, lot of research works on smart grid emerged, viz., [1]–[7]. Some of the existing literatures are discussed in this Section. Saad *et al.* [1] formulated a coalition game having multiple micro-grids, and proposed a distributed algorithm for forming the coalition. The authors [1] assumed that one micro-grid can exchange excess energy with the main grid. In case of power exchange between the micro-grid and the main grid, there will be loss of energy over the distribution line. Bakker *et al.* [2] proposed a distributed load management scheme with dynamic pricing strategy, and modeled it as a network congestion game. Nash equilibrium is presented in order to have an optimal solution. Misra *et al.* [3] proposed a distributed dynamic pricing mechanism (D2P) for charging PHEVs. They used two different pricing schemes such as home pricing scheme and roaming pricing scheme. Tushar *et al.* [6] studied how the PHEVs can be charged, when a single micro-grid has excess amount energy. They assumed that the smart grid is supported by a centralized system. However, they did not consider the distributed approach for charging and discharging the PHEVs in energy networks management. Saad *et al.* [7] proposed a game theoretic method for addressing various types of problems in smart grid. The authors conceptualized the future smart grid to encompass a large number of micro-grids. Hence, whenever some micro-grids have excess power, while other micro-grids have a scarcity of it, energy exchange takes place between them. However, the distributed energy networks management scheme using charging and discharging mechanism of the PHEVs is not considered.

The remainder of the paper is organized as follows. Section II describes the system model. In Section III, we formulate the game-theoretic electric vehicle energy networks management using non-cooperative Stackelberg game, and, thereafter, we discuss its properties. In this Section, we also propose the distributed algorithms, and discuss their performance in Section IV. Finally, we conclude the paper while citing few research directions in Section V.

II. System Model

We consider a green energy vehicular network (GEVN) consisting of a single micro-grid and several PHEVs. In GEVN, each PHEV either demands energy to the micro-grid or supplies excess energy to the micro-grid. We consider that the micro-grid serves a group of PHEVs within a coalition, i.e., a small geographical area, through electricity exchange stations (EESs), defined in Definition 1

Definition 1. *At electricity exchange station (EES), a PHEV either recharges its battery, i.e., charging, to go for a drive or sells energy to the micro-grid, i.e., discharging, in order to make profit. At the EES, each PHEV decides the strategy that it needs to choose from the available strategies — charging and discharging.*

We consider that within a small geographical area, $\mathcal{A}$, the micro-grid exchanges energy with the PHEVs, $\mathcal{N}(t)$, where $\mathcal{N}(t)$ is set of the available PHEVs at time slot $t \in \mathcal{T}$, and $\mathcal{T}$ is set of the time slots in a day. The schematic diagram of EVENT is shown in Figure 1. At time slot $t \in \mathcal{T}$, each PHEV $n \in \mathcal{N}(t)$ requests $d_n(t)$ amount of energy to the micro-grid to fulfill its energy requirement, when the micro-grid has $\mathcal{E}x(t)$ amount of excess energy after serving the residential customers. On the other hand, at time slot $\tau \in \mathcal{T}$, the micro-grid needs $\mathcal{R}e(\tau)$ amount of energy to fulfill the requirement of the residential customers, and the micro-grid requests the PHEVs $\mathcal{N}(\tau)$, where $\mathcal{N}(\tau)$ is set of the available PHEVs at time slot $\tau \in \mathcal{T}$. Each PHEVs $n \in \mathcal{N}(\tau)$, contributes $s_n(\tau)$ amount of energy to the micro-grid using EVENT to fulfill the energy requirement of the residential customers. Hence, the PHEVs and the micro-grid must satisfy the following constraints:

$$\mathcal{E}x(t) \geq \sum_{n=1}^{n \in \mathcal{N}(t)} d_n(t), \quad \text{if } \mathcal{E}x(t) > 0 \text{ and } \mathcal{R}e(t) < 0 \tag{1}$$

$$\mathcal{R}e(\tau) \leq \sum_{n=1}^{n \in \mathcal{N}(\tau)} s_n(\tau), \quad \text{if } \mathcal{R}e(\tau) > 0 \text{ and } \mathcal{E}x(\tau) < 0 \tag{2}$$

For charging at time slot t, the micro-grid needs to decide the price per unit energy, $p^{MG}(t)$, while the PHEVs need to satisfy the constraint given in Equation (1). Similarly, for discharging at time slot t, the PHEVs need to decide the price per unit energy, $p_n^{PHEV}(t)$, while satisfying the constraint in Equation (2). While deciding the price per unit energy, the micro-grid and the PHEVs must ensure that the price per unit energy is neither too high nor too low to avoid the less utilization of the excess energy, and the less revenue, respectively.

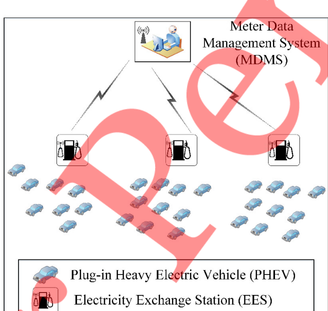


Fig. 1: Schematic diagram of EVENT

For proper green electric vehicle energy network management and successful energy trading, proper interaction between the micro-grid and the available PHEVs is needed. We divide the interactions into two types — *micro-grid to vehicle* (M2V) and *vehicle to micro-grid* (V2M) EVENT. Using M2V EVENT, each PHEV n decides the amount of required energy, $d_n(t)$, at time slot t, and the micro-grid needs to decide the price per unit energy, $p^{MG}(t)$. On the other hand, using V2M EVENT, each PHEV needs to decide the price per unit energy, $p_n^{PHEV}(t)$, and the micro-grid needs to decide the energy to be consumed, $s_n(t)$, from each PHEV n having excess amount of stored energy.

The proposed EVENT needs to satisfy constraints given in Equations (1) and (2). It is also to be noted that, in EVENT, the amount of energy to be consumed by each PHEV (in M2V) and the amount of energy to be supplied by each PHEV (in V2M) signify the satisfaction factor of each PHEV n, i.e., $sf_n^{PHEV}(t)$, and the satisfaction factor of the micro-grid, i.e., $sf^{MG}(t)$, defined in Definitions 2 and 3, respectively.

Definition 2. *In M2V EVENT, the satisfaction factor of each PHEV n, $sf_n^{PHEV}(t)$, is calculated with the ratio of the residual energy stored, $\mathcal{E}_n^{res}(t)$, at the PHEV-end, and the maximum energy storage capacity, $\mathcal{E}_n^{max}$, of each PHEV n. Therefore,*

$$sf_n^{PHEV}(t) = \frac{\mathcal{E}_n^{res}(t)}{\mathcal{E}_n^{max}}, \quad \forall n \in \mathcal{N}(t) \tag{3}$$

Definition 3. *In V2M EVENT, the satisfaction factor of the micro-grid, $sf^{MG}(t)$, is calculated with the ratio of the amount of consumed energy by the micro-grid, $\mathcal{G}_{MG}^{con}(t)$, and the amount of energy requested by the micro-grid, $\mathcal{G}_{MG}^{req}(t)$. Mathematically,*

$$sf^{MG}(t) = \frac{\mathcal{G}_{MG}^{con}(t)}{\mathcal{G}_{MG}^{req}(t)} \tag{4}$$

Thus, the main challenges, faced to develop the EVENT that manages the energy transfer between the micro-grid and the PHEVs, and vice-versa are as follows:

(i) Modeling the energy management scheme, i.e., EVENT, and the interaction between the micro-grid and the PHEVs.

(ii) Developing algorithms for the micro-grid to decide the price per unit energy using M2V EVENT, and the amount of energy to be consumed using V2M EVENT.

(iii) Developing algorithms for each PHEV to decide the amount of required energy using M2V EVENT, and the price per unit energy using V2M EVENT.

Communication between the micro-grid and the PHEVs: We assume that the communication networking model between the micro-grid and the PHEVs is wireless mesh network (WMN). We use the IEEE 802.11b protocol for the communication between the micro-grid and the PHEVs. In M2V EVENT, each PHEV sends message to the micro-grid having information of the amount of required energy for charging, as shown in Figure 2(a). Based on that information, the micro-grid decides the price per unit energy and sends an acknowledgment message, as shown in Figure 2(b).

MsgType	MsgSeqNum	PHEV_ID	Req_Energy

(a) Request message of PHEV

MsgType	MsgSeqNum	AckFlag	Price

(b) Acknowledgement message of micro-grid

Fig. 2: Message format in M2V EVENT

In V2M EVENT, initially, the micro-grid broadcasts the total energy requirement, as shown in Figure 3(a). Hence, the

MsgType	MsgSeqNum	MsgID	Req_Energy

(a) Query message of micro-grid

MsgType	MsgSeqNum	AckFlag	ExcessEnergy

(b) Acknowledge message of PHEV

MsgType	MsgSeqNum	PHEV_ID	ReqEnergy

(c) Request message of micro-grid

MsgType	MsgSeqNum	AckFlag	Price

(d) Reply message of PHEV

Fig. 3: Message format in V2M EVENT

PHEVs having excess amount of stored energy send acknowledgment messages, as shown Figure 3(b). Now, the micro-grid unicasts message with the information of the required amount of energy, and the PHEVs send acknowledgment messages with the decided price per unit energy, as shown in Figures 3(c) and 3(d), respectively.

III. Proposed Electric Vehicle Energy Networks Management Game

A. Game Formulation

To study the interaction between the micro-grid and the PHEVs in green electric vehicle energy networks management (EVENT), we use a *non-cooperative single leader multiple follower Stackelberg game* [8] theoretic approach. In EVENT, the micro-grid and the PHEVs form the set of the players. In this paper, we consider that the micro-grid acts as the leader, and the PHEVs act as the followers. Based on the decision taken by the micro-grid, the PHEVs decide their respective strategies using non-cooperation. Using M2V EVENT, the micro-grid decides the price per unit energy, whereas the PHEVs decide the amount of energy to be consumed for storage. On the other hand, using V2M EVENT, the micro-grid decides the amount of energy to be requested to each PHEV, and each PHEV decides the price per unit energy based on the amount of requested energy by the micro-grid. The components of overall are as follows:

(i) At time slot $t \in \mathcal{T}$, using M2V EVENT, each PHEV $n \in \mathcal{N}(t)$ demands $d_n(t)$ amount of energy, when the micro-grid has generated $\mathcal{E}x(t)$ amount of excess energy.

(ii) Using M2V EVENT, the micro-grid decides the price per unit energy, $p^{MG}(t)$, based on the amount of excess energy generated, i.e., $\mathcal{E}x(t)$.

(iii) Using V2M EVENT, each PHEV $n \in \mathcal{N}(t)$ supplies $s_n(t)$ amount of energy to fulfill $\mathcal{R}e(t)$ amount of energy deficiency of the micro-grid.

(iv) Using V2M EVENT, the micro-grid pays $p_n^{PHEV}(t)$ price per unit energy to PHEV n for supplying $s_n(t)$ amount of energy.

We divide the game formulation in two types — M2V EVENT and V2M EVENT. The game formulation using M2V EVENT and V2M EVENT are discussed in Sections III-A1 and III-A2, respectively.

1) Game Formulation using M2V EVENT: Using M2V EVENT, each PHEV n decides the amount of energy to be consumed, $d_n(t)$, at time slot t, and the micro-grid decides the price per unit energy, $p^{MG}(t)$, for that time slot. On the other hand, using V2M EVENT, the micro-grid decides the amount of energy to be requested, $s_n(t)$, to each PHEV n having excess amount of energy. The utility function for the PHEVs and the micro-grid are discussed in the following sections.

a) Utility function for PHEV using M2V EVENT: In M2V EVENT, for each PHEV $n \in \mathcal{N}(t)$, we define the utility function $\mathcal{U}_n(\cdot)$ to represent the satisfaction by consuming $b_n(t)$ amount of energy at time slot t. We define the rules for utility calculation of each PHEV n as follows:

(i) The utility function of PHEV n, $\mathcal{U}_n(\cdot)$, is considered to be non-decreasing, as with increase in amount of consumed energy $d_n(t)$, i.e., $\hat{d}_n(t) = \tilde{d}_n(t) - d_n(t)$, the residual energy of each PHEV n, i.e., sf_n^{PHEV}, becomes higher. Here, $\tilde{d}_n(t)$ and $d_n(t)$ are the new and current value of requested energy by the PHEV n to the micro-grid. Mathematically,

$$\frac{\delta \mathcal{U}_n(\cdot)}{\delta \hat{d}_n(t)} \geq 0 \tag{5}$$

(ii) The marginal utility of each PHEV $n, \mathcal{U}_n(\cdot)$, is considered to be decreasing, as with the increase in energy consumption after reaching equilibrium state [9], the PHEVs will be over powered and that may lead to an accident. Mathematically,

$$\frac{\delta^2 \mathcal{U}_n(\cdot)}{\delta \hat{d}_n(t)^2} < 0 \tag{6}$$

(iii) The amount of consumed energy by each PHEV n reduces with increase in price per unit energy decided by the micro-grid, $p^{MG}(t)$. Therefore,

$$\frac{\delta \mathcal{U}_n(\cdot)}{\delta p^{MG}(t)} < 0 \tag{7}$$

Therefore, for each PHEV n, we define the revenue function, $\mathcal{R}_n(\cdot)$, and the cost function, $\mathcal{C}_n(\cdot)$, in Definitions 4 and 5, respectively. We define the utility function, $\mathcal{U}_n(\cdot)$, of each PHEV n as the difference of revenue function, $\mathcal{R}_n(\cdot)$, and the cost function, $\mathcal{C}_n(\cdot)$. Mathematically,

$$\mathcal{U}_n(\cdot) = \mathcal{R}_n(\cdot) - \mathcal{C}_n(\cdot), \quad \forall n \in \mathcal{N}(t) \tag{8}$$

Definition 4. *The revenue function, $\mathcal{R}_n(\cdot)$, of each PHEV n is considered to be a concave function. Therefore, considering $\hat{d}_n(t) = \tilde{d}_n - d_n$, we define $\mathcal{R}_n(\cdot)$ as follows:*

$$\mathcal{R}_n(\cdot) = \mathcal{E}_n^{max} \tan^{-1}\left(e^{-\frac{\hat{d}_n(t)}{d_n(t)}}\right) \tag{9}$$

Definition 5. *The cost function, $\mathcal{C}_n(\cdot)$, of each PHEV n is considered to have linear price coefficient, i.e., ratio of the profit per unit energy decided by the micro-grid, i.e., $p^{MG}(t) - c^{MG}(t)$, and the generation cost per unit energy, $c^{MG}(t)$. Therefore, we define $\mathcal{C}_n(\cdot)$ as follows:*

$$\mathcal{C}_n(\cdot) = \left(\frac{p^{MG}(t) - c^{MG}(t)}{c^{MG}(t)}\right)\tilde{d}_n \tag{10}$$

Hence, we get the utility function of PHEV n as follows:

$$\mathcal{U}_n(\cdot) = \mathcal{E}_n^{max} \tan^{-1}\left(e^{-\frac{\tilde{d}_n - d_n}{d_n(t)}}\right) - \left(\frac{p^{MG}(t) - c^{MG}(t)}{c^{MG}(t)}\right)\tilde{d}_n \tag{11}$$

b) Utility function for micro-grid using M2V EVENT: In M2V EVENT, the micro-grid makes profit by supplying the excess amount of generated energy, $\mathcal{E}x(t)$, to the connected PHEVs, i.e., $\mathcal{N}(t)$, having price per unit energy, $p^{MG}(t)$. Therefore, the profit of the micro-grid, $\mathcal{P}^n_{MG}(t)$, by selling $\tilde{d}_n(t)$ amount of energy to each PHEV n is as follows:

$$\mathcal{P}^n_{MG}(t) = p^{MG}(t)\tilde{d}_n(t) \tag{12}$$

Hence, total profit of the micro-grid at time slot t, i.e., the utility function of the micro-grid, $\mathcal{P}_{MG}(t)$, using M2V EVENT, is calculated as:

$$\mathcal{P}_{MG}(t) = \sum_{n=1}^{n\in\mathcal{N}(t)} \mathcal{P}^n_{MG}(t) \tag{13}$$

Therefore, in M2V EVENT, the micro-grid tries to maximize its utility function to maximize its profit.

2) Game Formulation using V2M EVENT: Using V2M EVENT, the micro-grid consumes the deficient amount of energy, $\mathcal{R}e(t)$, from the available PHEVs. In V2M EVENT, the mciro-grid decides the energy to be consumed, $s_n(t)$, from each PHEV n having excess stored energy. The PHEVs supply energy to the micro-grid with the price per unit energy, $p_n^{PHEV}(t)$, decided by the PHEV n.

a) Utility function for PHEV using V2M EVENT: In V2M EVENT for each PHEV n, we define the utility function, $\chi_n(\cdot)$, to represent the profit of each PHEV n by supplying the excess amount of energy, $s_n(t)$, with $p_n^{PHEV}(t)$ price per unit energy. Hence, we define the utility function of the PHEV $n \in \mathcal{N}(t)$, $\chi_n(\cdot)$, in V2M EVENT as follows:

$$\chi_n(\cdot) = p_n^{PHEV}(t)s_n(t), \quad \forall n \in \mathcal{N}(t) \tag{14}$$

b) Utility function for micro-grid using V2M EVENT: In V2M EVENT, the utility function of the micro-grid, $\psi(\cdot)$, represents the satisfaction of the micro-grid by consuming $s_n(t)$ amount of energy from PHEV $n \in \mathcal{N}(t)$. The net utility function $\psi(\cdot)$ considers to be sum of the individual utility value by consuming $s_n(t)$ amount of energy from PHEV n, i.e., $\psi_n(\cdot)$, with unit weight. Mathematically,

$$\psi(\cdot) = \sum_{n=1}^{n\in\mathcal{N}(t)} \psi_n(\cdot) \tag{15}$$

We define the rules of utility calculation of the micro-grid as follows:

(i) The individual utility of the micro-grid $\psi_n(\cdot)$ is considered to be non-decreasing, as with the increase in energy consumption of the micro-grid $s_n(t)$, i.e., $\hat{s}_n(t) = \tilde{s}_n(t) - s_n(t)$, the satisfaction factor of the micro-grid, $sf^{MG}(t)$, increases, and the residential customers and PHEVs connected with the micro-grid get high quality of energy service. Here, $\tilde{s}_n(t)$ and $s_n(t)$ are the new and current value of the requested amount of energy to each PHEV n by the micro-grid. Mathematically,

$$\frac{\delta\psi_n(\cdot)}{\delta\tilde{s}_n(t)} \geq 0 \tag{16}$$

(ii) The marginal utility of utility function, $\psi_n(\cdot)$, is considered to be decreasing, as with the increase in energy supply, $\tilde{s}_n(t)$, after reaching Nash equilibrium, the supplied energy to the micro-grid is not properly utilized, and the satisfaction factor of PHEV n, $sf_n^{PHEV}(t)$ reduces. Therefore,

$$\frac{\delta^2\psi_n(\cdot)}{\delta\tilde{s}_n(t)^2} < 0 \tag{17}$$

(iii) The amount of energy to be consumed by the micro-grid from each PHEV n varies inversely with the price per unit energy decided by each PHEV n, i.e., $p_n^{PHEV}(t)$. Mathematically,

$$\frac{\delta\psi_n(\cdot)}{\delta p_n^{PHEV}(t)} < 0 \tag{18}$$

Therefore, for consuming $\tilde{s}_n(t)$ amount of energy from PHEV $n \in \mathcal{N}(t)$, we define the revenue function, $\mathbb{R}_n(\cdot)$, and the cost function, $\mathbb{C}_n(\cdot)$, of the micro-grid in Definitions 6 and 7, respectively. We define the utility function of the micro-grid for PHEV n, i.e., $\psi_n(\cdot)$, as difference between the revenue function and the cost function of the micro-grid. Mathematically,

$$\psi_n(\cdot) = \mathbb{R}_n(\cdot) - \mathbb{C}_n(\cdot), \quad \forall n \in \mathcal{N}(t) \tag{19}$$

Definition 6. *The revenue function, $\mathbb{R}_n(\cdot)$, of the micro-grid is considered to be a concave function. Hence, we define $\mathbb{R}_n(\cdot)$ as follows:*

$$\mathbb{R}_n(\cdot) = \mathcal{R}e(t)\tan^{-1}\left(e^{-\frac{\hat{s}_n(t)}{s_n(t)}}\right) \tag{20}$$

where $\hat{s}_n(t) = \tilde{s}_n(t) - s_n(t)$.

Definition 7. *The cost function, $\mathbb{C}_n(\cdot)$, of the micro-grid is defined as multiplication of the amount of energy consumed from PHEV n, $\tilde{s}_n(t)$, and the price coefficient. The price coefficient is defined as the ratio of the profit per unit energy decided by PHEV n, i.e., $p_n^{PHEV}(t) - p^{MG}(\tau)$, and the price per unit consumed energy, i.e., $p^{MG}(\tau)$, where $\tau < t$. Mathematically,*

$$\mathbb{C}_n(\cdot) = \left(\frac{p_n^{PHEV}(t) - p^{MG}(\tau)}{p^{MG}(\tau)}\right)\tilde{s}_n(t), \quad \tau < t \tag{21}$$

Hence, we get the utility function, $\psi_n(\cdot)$, of the micro-grid as follows:

$$\psi_n(\cdot) = \mathcal{R}e(t)\tan^{-1}\left(e^{-\frac{\hat{s}_n(t)}{s_n(t)}}\right) - \left(\frac{p_n^{PHEV}(t) - p^{MG}(\tau)}{p^{MG}(\tau)}\right)\tilde{s}_n(t) \tag{22}$$

B. Existence of Nash Equilibrium

In this section, we determine the existence of Nash equilibrium [9] of M2V EVENT and V2M EVENT in Theorems 1 and 2.

Theorem 1. *In M2V EVENT, given a fixed price per unit energy decided by the micro-grid, $p^{MG}(t)$, there exists a generalized Nash equilibrium (GNE), as there exists a variational equilibrium for the utility function, $\mathcal{U}_n(\cdot)$, for each PHEV n. The condition for GNE is as follows:*

$$\mathcal{U}_n(\tilde{d}_n^*(t), p^{MG*}(t)) \geq \mathcal{U}_n(\tilde{d}_n(t), p^{MG*}(t)) \tag{23}$$

Proof: The utility function of each PHEV n, $\mathcal{U}_n(\cdot)$, is needed to be maximized. Hence, applying the *Karush-Kuhn-Tucker* condition, we get:

$$\nabla_n\mathcal{U}_n(\cdot) - \nabla_n\left(\sum \tilde{d}_n(t) - \mathcal{E}x(t)\right)\lambda_n = 0, \quad \lambda_n \geq 0,$$
$$\nabla_n\left(\sum \tilde{d}_n(t) - \mathcal{E}x(t)\right)\lambda_n = 0, \tag{24}$$

Using *variational inequality* principle, we consider the utility matrix, $\boldsymbol{\mathcal{U}}(t) = [\mathcal{U}_1(\cdot);\mathcal{U}_2(\cdot);\cdots;\mathcal{U}_n(\cdot);\cdots;\mathcal{U}_{|\mathcal{N}(t)|}(\cdot)]$, having Lagrangian multipliers for all the followers, i.e., λ_n, $\forall n \in \mathcal{N}(t)$, are same. Hence, taking derivative of the *Jacobian matrix* of $\boldsymbol{\mathcal{U}}(t)$, we get a diagonal negative matrix. Therefore, we conclude that there exists GNE using M2V EVENT. ∎

Theorem 2. *In V2M EVENT, given a fixed price per unit energy by each PHEV n, $p_n^{PHEV}(t)$, there exists a generalized Nash equilibrium (GNE), as there exists a variational equilibrium for utility function, $\psi(\cdot)$, of the micro-grid. The condition for GNE is as follows:*

$$\psi(\tilde{s}_n^*(t), p_n^{PHEV*}(t)) \geq \psi(\tilde{s}_n(t), p_n^{PHEV*}(t)) \tag{25}$$

Proof: The utility function of the micro-grid, $\psi(\cdot)$, is needed to be maximized. Hence, applying the *Karush-Kuhn-Tucker* condition, we get:

$$\nabla\psi(\cdot) - \nabla(\sum \tilde{s}_n(t) - \mathcal{R}e(t))\mu = 0, \quad \mu \geq 0,$$
$$\nabla(\sum \tilde{s}_n(t) - \mathcal{R}e(t))\mu = 0, \tag{26}$$

Again using *variational inequality* principle, we consider the *Jacobian* matrix of the utility function, $\psi(t)$. Hence, taking derivative of the Jacobian matrix of $\psi(t)$, we get a diagonal negative matrix. Therefore, we conclude that there exists GNE using V2M EVENT. ∎

C. Algorithms

For green electric vehicle energy networks management (EVENT), we propose two different algorithms — M2V EVENT and V2M EVENT, as shown in Algorithms 1 and 2, respectively. In M2V EVENT algorithm, the micro-grid decides the price per unit energy, $p^{MG}(t)$, and each PHEV decides the amount of energy required for charging, $\tilde{d}_n(t)$. On the other hand, in V2M EVENT algorithm, the micro-grid decides the amount of energy to be requested to each PHEV n, $\tilde{s}_n(t)$, and each PHEV decides the price per unit energy for discharging, $p_n^{PHEV}(t)$.

Algorithm 1: M2V EVENT Algorithm

```
M2V EVENT for PHEV
    Inputs : p^{MG}(t), c^{MG}(t), E_n^{max}, d_n(t),
             α_n: Increment factor in requested energy per iteration
    Output: d̃_n(t)
    d̃_n(t) = (1 + α_n) d_n(t)
    if U_n(d̃_n(t), p^{MG}(t)) ≥ U_n(d_n(t), p^{MG}(t)) then
        // U_n is to be calculated using Equation (11)
        d_n(t) = d̃_n(t)
    else
        d̃_n(t) = d_n(t)            // Nash equilibrium reached
    end
    Send request message with d̃_n(t)
    return
M2V EVENT for micro-grid
    Inputs : d̃_n(t), p^{MG}(t),
             β: Increment factor in price per unit energy in each iteration
    Output: p^{MG*}(t): New price per unit energy decided by the micro-grid
    p^{MG*}(t) = (1 + β) p^{MG}(t)
    if P^n_{MG}(d̃_n(t), p^{MG*}(t)) ≥ P^n_{MG}(d̃_n(t), p^{MG}(t)) then
        p^{MG}(t) = p^{MG*}(t)
        // p^{MG}(t) is the current value of price per unit
        energy
    else
        p^{MG*}(t) = p^{MG}(t)       // Nash equilibrium reached
    end
    Send acknowledgment message with p^{MG*}(t)
    return
```

Algorithm 2: V2M EVENT Algorithm

```
V2M EVENT for micro-grid
    Inputs : p_n^{PHEV}(t), p^{MG}(τ), Re(t), s_n(t),
             γ: Increment factor in requested energy per iteration
    Output: s̃_n(t)
    s̃_n(t) = (1 + γ) s_n(t)
    if ψ_n(s̃_n(t), p_n^{PHEV}(t)) ≥ ψ_n(s_n(t), p_n^{PHEV}(t)) then
        // ψ_n is to be calculated using Equation (22)
        s_n(t) = s̃_n(t)
        // s̃_n(t) is the current value of energy to be
        requested
    else
        // Nash equilibrium reached
        s̃_n(t) = s_n(t)
    end
    Send request message with s̃_n(t)
    return
V2M EVENT for PHEV
    Inputs : s̃_n(t), p_n^{PHEV}(t),
             η_n: Increment factor in price per unit energy in each iteration
    Output: p_n^{PHEV*}(t)
    p_n^{PHEV*}(t) = (1 + η_n) p_n^{PHEV}(t)
    if χ_n(s̃_n(t), p_n^{PHEV*}(t)) ≥ χ_n(s̃_n(t), p_n^{PHEV}(t)) then
        p_n^{PHEV}(t) = p_n^{PHEV*}(t)
        // p_n^{PHEV}(t) is the current value of price per
        unit energy
    else
        // Nash equilibrium reached
        p_n^{PHEV*}(t) = p_n^{PHEV}(t)
    end
    Send acknowledgment message with p_n^{PHEV*}(t)
    return
```

IV. Performance Evaluation

A. Simulation Parameters

To evaluate the performance of the proposed scheme, EVENT, we considered randomly generated position for the PHEVs on MATLAB simulation platform. In this work, we took randomly generated value for energy consumption profile of the PHEVs, as shown in Table I.

TABLE I: Simulation Parameters

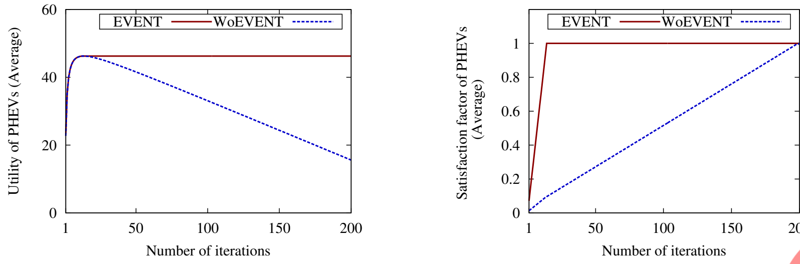
Parameter	Value
Simulation area	20×20 km^2
Number of PHEVs	1500
Requested energy by PHEVs	35-65 kWh
Requested energy by micro-grid	30-40 MWh
Excess energy generated	99 MWh
Excess energy stored by PHEVs	30-50 kWh
Generation cost	17 USD/MWh

B. Benchmark

The performance of proposed scheme for green electric vehicle energy network management (EVENT) is evaluated by comparing it with other energy management policy, i.e., without game-theoretic electric vehicle energy network management (WoEVENT). Using WoEVENT, in M2V, the PHEVs consumes high amount of energy until the batteries get fully charged. On the other hand, in V2M, the PHEVs do not discharge their batteries below 80% of the capacity of their batteries. We refer to these different energy management policies as EVENT, and WoEVENT, through the rest of the paper. We show that the EVENT performs better than WoEVENT.

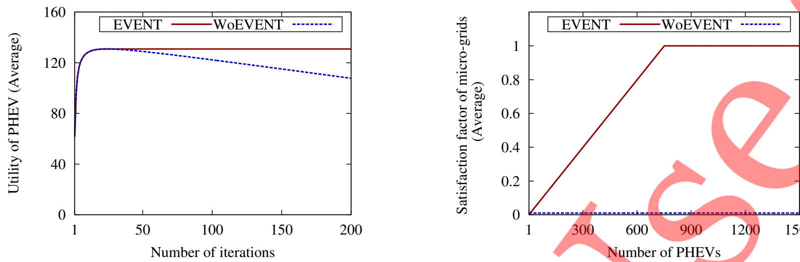
C. Performance Metrics

(i) *Price per unit energy using M2V EVENT*: The price per unit energy using M2V EVENT is decided by the micro-



(a) Utility value of PHEVs (b) Satisfaction factor of PHEVs

Fig. 4: Performance evaluation using M2V EVENT



(a) Utility value of PHEVs (b) Satisfaction factor of micro-grid

Fig. 5: Performance evaluation using V2M EVENT

grid based on the real-time communication with the PHEVs. The price may vary with the varying energy request by the PHEVs.

(ii) *Satisfaction factor of each PHEV*: In M2V EVENT, each PHEV consumes the amount of required energy that can be used for driving, or in V2M EVENT for discharging. If the consumed energy is high, the satisfaction factor of the PHEV is high.

(iii) *Price per unit energy using V2M EVENT*: The price per unit energy decided by each PHEV depends on the amount of energy requested by the micro-grid for discharging, and the previous price per unit energy during charging.

(iv) *Satisfaction factor of the micro-grid*: In V2M EVENT, the satisfaction factor of the micro-grid is calculated using the ratio of the amount of energy consumed and the amount of energy requested by the micro-grid.

D. Results and Discussions

For the sake of simulation, we assume that micro-grid calculates the real-time supply and demand in every 5 seconds interval. In Figure 4(a), the utility of PHEV signifies that the energy demand reaches generalized Nash equilibrium within a few iteration, and does not deviate from generalized Nash equilibrium. However, there is no stable solution using WoEVENT. Figure 4(b) signifies that the PHEV gains its maximum satisfaction factor within a few iterations.

Using V2M EVENT, Figure 5(a) shows that there exists generalized Nash equilibrium using EVENT, whereas using WoEVENT, no equilibrium solution can be derived. From Figure 5(b), we conclude that with increase in number of PHEVs, the micro-grid reaches to the maximum satisfaction factor of the micro-grid. With more increase in number of the PHEVs, the satisfaction factor of the micro-grid remains constant.

V. Conclusion

In this paper, we formulated a non-cooperative Stackelberg game theoretic approach to study the problem of green electric vehicle energy networks management in smart grid. Based on the proposed scheme, EVENT, we showed how the vehicular energy network management can be performed using M2V EVENT and V2M EVENT for charging and discharging, respectively. The simulation results show that the proposed approach yields improved results.

Future extension of this work includes understanding how the energy can be transferred from one place to another place using the movement of the PHEVs, so that energy can reach some remote places, and enlighten the lands with electricity.

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