Distributed Home Energy Management System with Storage in Smart Grid Using Game Theory

Ayan Mondal, *Student Member, IEEE*, Sudip Misra, *Senior Member, IEEE*, and
Mohammad S. Obaidat, *Fellow, IEEE*, & *Fellow, SCS*

***Abstract*—In this paper, the problem of distributed home energy management system with storage in a coalition, which consists of multiple micro-grids and multiple customers, is studied using the *multiple leader multiple follower Stackelberg game* theoretic model — a multi stage, and multi level game. The micro-grids, which act as the leaders, need to decide the minimum amount of energy to be generated with the help of central energy management unit (CEMU), and the optimum price per unit energy to maximize their profit. On the other hand, the customers, which act as the followers, need to decide the optimum amount of energy to be consumed, including the energy to be requested for storage. Using the proposed distributed scheme, i.e., *home energy management system with storage* (*HoMeS*), the earned-profit of the grid improves up to 55%, and the customers consume almost 30.79% higher amount of energy, which, in turn, increases the utilization of the generated energy by the micro-grids.**

***Index Terms*—Energy Management, Storage, Extensive game, Multiple Leader Multiple Follower Stackelberg Game, Micro-Grid, Smart Grid.**

I. INTRODUCTION

To achieve high reliability in power systems, traditional electrical grids need to be designed as modernized electrical systems, termed as *smart grids*. A smart grid [1]–[3] is visualized to be a cyber-physical system equipped with sustainable models of energy production, distribution, and usage [4]. It also integrates several advanced techniques such as advanced metering infrastructure (AMI), automatic meter reading (AMR), distributed energy resources (DER), energy management systems (EMS), intelligent electronic devices (IEDs), and plug-in hybrid electrical vehicles (PHEVs) [2]. Unlike in existing power systems, in which electricity is distributed unidirectionally to the customers by the main grid having a centralized system, in a smart grid with duplex communication infrastructure, the large scale traditional electrical grid is divided into micro-grids [5] having bi-directional electricity exchange facility with the substation, and the main grid. In the presence of several micro-grids, it is desirable to allow group of micro-grids to service a group of customers based on their demands in a distributed manner, so as to relax the load on the main grid. One of the important features in a smart grid is the demand-side energy distribution, which gives the opportunity for flexible energy demand according to the requirements of the customers.

The micro-grids generate energy using renewable energy resources such as wind power, solar energy [3], and hydro power [6]. So, the amount of generated energy is not fixed at different times in a day. If the total energy demand by the customers to the micro-grid exceeds the total generated energy by that micro-grid, the micro-grid requests the main grid to supply the deficient amount of energy to fulfill the requirements of the customers. As the requested energy by the customers to each micro-grid is discreet, the load on each micro-grid does not remain the same in any specific time. During on-peak hours, the demand of the customers is higher than the demand during off-peak hours. So, in on-peak hours, the micro-grids request the main grid to supply energy to fulfill the customers' demand, whereas in off-peak hours, the micro-grids have excess amount of energy. In such a condition, the existence of storage capacity with the customers will be cost effective, and the reliability of the energy supply will also increase. Additionally, having storage facility at the customer-end, in on-peak hours, the amount of requested energy by the customers will be reduced while the required energy can be served using stored energy. On the other hand, in off-peak hours, the customers consume high amount of energy including energy for storage. Moreover, we consider that each customer can communicate with multiple micro-grids available with a coalition, in order to reduce energy loss. Consequently, each customer needs to decide the optimal amount of energy to be requested to micro-grid in order to maximize its utility. On the other hand, the micro-grids decide the amount of energy that has to be generated by each micro-grid, and the price per unit energy to maximize the payoff value corresponding to the utility function, and, consequently, the profit.

In this paper, we introduce a *game theoretic approach* for distributed home energy management system with storage (HoMeS). We use a multiple leader multiple follower Stackelberg game to decide the strategies for the micro-grids to maximize their profit and proper utilization of generated energy, and the strategies for the customers, so as to fulfill their energy requirement by maximizing their individual pay-off values. Based on the remaining stored energy, each customer n decides the required energy for the appliances, which is the minimum amount of requested energy for the customer n, and broadcasts that information within the coalition. On receiving these informations from the customers, the micro-grids decide the minimum energy to be generated and minimum price per unit energy based on the cost of generation per unit energy. The micro-grid broadcasts the price per unit energy. Each customer decides the amount of energy to be requested, including the amount of energy for storage for future use.

Manuscript received...

A. Mondal and S. Misra are with the School of Information Technology, Indian Institute of Technology Kharagpur, India (Email: ayanmondal@sit.iitkgp.ernet.in; smisra@sit.iitkgp.ernet.in).

Mohammad S. Obaidat is with the Department of Computer Science and Software Engineering Monmouth University, West Long Branch, NJ, USA (Email: obaidat@monmouth.edu).

Each micro-grid m decides the price per unit energy based on the amount of requested energy to the micro-grid m using a non-cooperative approach. In summary, our contribution in this paper as follows:

a) We present the *home energy management system with storage* (*HoMeS*) model for real-time energy consumption of customers in the presence of storage facilities and several micro-grids in a coalition.

b) The multiple leader multiple follower Stackelberg game theoretic approach is used to evaluate the optimal strategies of the micro-grids using a cooperative game, which is the initial phase of the proposed game, and the optimal strategies of the customers using a non-cooperative game, which is the next phase of the proposed game.

c) We present three different algorithms. The first algorithm is used in the *Initialization Phase* for the micro-grids to determine the minimum amount of energy to be generated. The second algorithm is used by the customers to decide the amount of requested energy based on the real-time price of energy. In the final proposed algorithm, the micro-grids decide the price per unit energy on a real-time basis depending on the total amount of requested energy.

The remainder of the paper is organized as follows. We briefly present the related literature in Section II. Section III describes the system model. In Section IV, we formulate the game theoretic method using the multiple leader multiple follower Stackelberg game, and, thereafter, we discuss its properties. In this Section, we also propose the distributed algorithms, and discuss their performance in Section V. Finally, we conclude the paper while citing few research directions in Section VI.

II. LITERATURE REVIEW

In the last few years, lot of research work on smart grid emerged, viz., [7]–[18]. Some of the existing literature are discussed in this Section. Saad *et al.* [7], [8] formulated a coalition game having multiple micro-grids, and proposed a distributed algorithm for forming the coalition assuming that one micro-grid can exchange excess energy with the main grid [7] or other micro-grids having deficiency of energy [8]. In case of power exchange between the micro-grid and the main grid, there will be loss of energy over the distribution line. Bakker *et al.* [12] proposed a distributed load management system with dynamic pricing strategy, and have modeled it as a network congestion game. Misra *et al.* [14] proposed a distributed dynamic pricing mechanism (D2P) for charging PHEVs. They used two different pricing schemes such as home pricing scheme and roaming pricing scheme. Molderink *et al.* [15] proposed an algorithm by using the energy in the off-peak, and the on-peak hours, with a virtual power plant, for energy management system. However, none of these work consider the storage issue in the customer side.

Fang *et al.* [17] proposed different energy management schemes. However, in this work, new opportunities for more improved residential energy management system and bill reduction are studied without considering the impact of stored energy on the customers. Erol-Kantarci and Mouftah [18] proposed a time-to-use (TOU) aware-energy management scheme. In this scheme, a customer consumes energy according to the time, i.e., an on-peak hour or an off-peak hour. In the on-peak hour, the customer has to wait for being served. Otherwise, the customer demands the required energy without waiting, if the delay is greater than the maximum allowable delay. Yet, the energy management policy adopted by the customers and the micro-grids need further research to have an optimal solution and with minimum delay and less message overhead.

In the existing literature, several energy generation and consumption models are also proposed, by considering different uncertainties which impose imbalance costs to the system operators [19]. Some of the existing literature are discussed here. Soroudi *et al.* [20], [21] studied a Monte Carlo simulation based probabilistic dynamic model for multi-objective distributed generation considering uncertainties – load distribution, generated power, and price. They provided Pareto optimal solution for cost optimization and technical risk optimization. Soroudi *et al.* [22], [23] evaluated the effect of renewable distributed generation units on active losses and distribution network in load supply with uncertainties using a fuzzy evaluation tool [22] and operation of distributed generation units on a distribution networks [23]. However, none of these works considers the uncertainty of energy distribution where energy storage facility is available at the customer's ends.

In contrast to the existing works, a multi-stage stochastic game theoretic model is used in this paper to characterize the effect of storage with the customers in the smart grid. We use the multiple leader multiple follower Stackelberg game to develop the optimal solution for home energy management system for each customer.

III. SYSTEM MODEL

We consider a energy distribution system consisting of multiple micro-grids and multiple customers. The schematic diagram of an energy management system is given in Figure 1. In this, each customer has a smart meter and a communication unit. We consider a group of customers connected to a single micro-grid. The total charging period in a day is divided into multiple time slots, T. In each time slot $t \leq T$, each micro-grid, $m \in \mathcal{M}$, where $\mathcal{M}$ the set of micro-grids, has to decide the amount of energy to be generated G_m^t for selling to the connected customers to meet their energy demand and maximizing its own revenue. The total energy generated in time slot t and the total energy generated by each micro-grid

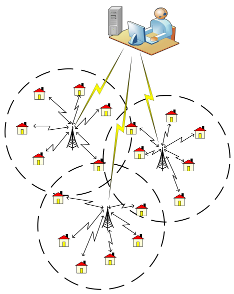
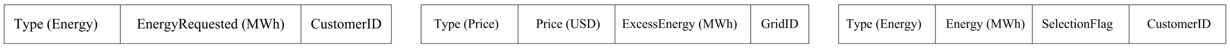


Fig. 1: Schematic Diagram of the Energy Management System



(a) Request message in IP (b) Reply message from micro-grid (c) Request message in FP

Fig. 2: Message Formats in HoMeS

$m \in \mathcal{M}$ in a day are denoted as G^t and G_m, respectively. Mathematically,

$$G^t = \sum_{m=1}^{m \in \mathcal{M}} G_m^t \text{ and } G_m = \sum_{t=1}^{T} G_m^t \tag{1}$$

A group of micro-grids $\mathcal{W} \subseteq \mathcal{M}$ form a coalition Co_w, where $w \in (0, \frac{|\mathcal{M}|}{|\mathcal{W}|}]$, and serve a small geographical area, $\mathcal{A}_w$, consisting of a group of customers $\mathcal{C}_w \subseteq \mathcal{N}$, where $\mathcal{N}$ is the set of N number of customers, and $\mathcal{C}_w$ is the set of customers under coalition Co_w. Within a coalition, the micro-grids can exchange energy between themselves.

Each customer $n \in \mathcal{N}$ requests a certain amount of energy e_n from its service provider, i.e., the corresponding micro-grid, to fulfill its energy requirement, i.e., the energy requirement for the appliances of the customer n, a_n, and energy requirement for storage, x_n. Therefore,

$$e_n = a_n + x_n, \qquad \forall n \in \mathcal{N} \tag{2}$$

The demanded energy, e_n, of customer n may vary in different time slots, as the energy requirement of a customer n is based on different parameters such as the maximum storage capacity, E_{max}; the amount of remaining stored energy, E_{res}; the price per unit energy decided by the service provider; the energy required for daily appliances, a_n; and the energy required for storage, x_n. We assume that the energy requirement for daily appliances, a_n, is known to the micro-grids on a day-ahead basis, and the micro-grid has to supply a_n amount of required energy. Therefore, in a coalition Co_w having $\mathcal{W}$ micro-grids, the total amount of energy has to be generated is at least $\sum_{n=1}^{n \in \mathcal{C}_w} a_n$ amount of energy. Mathematically,

$$\left[\arg\min \sum_{m=1}^{m \in \mathcal{W}} G_m \geq \sum_{n=1}^{n \in \mathcal{C}_w} a_n\right] \text{ and } \left[\sum_{m=1}^{m \in \mathcal{W}} G_m \geq \sum_{n=1}^{n \in \mathcal{C}_w} e_n\right] \tag{3}$$

Hence, the net available energy for storage $\mathcal{S}_w$ in a coalition Co_w having $\mathcal{W}$ micro-grids is given by:

$$\mathcal{S}_w = \left(\sum_{m=1}^{m \in \mathcal{W}} G_m - \sum_{n=1}^{n \in \mathcal{C}_w} a_n\right) \tag{4}$$

Since the net available energy $\mathcal{S}_w$ is fixed for the customers, the demands for storage of a customer n, i.e., x_n, has to satisfy the following condition:

$$\sum_{n=1}^{n \in \mathcal{C}_w} x_n \leq \mathcal{S}_w \tag{5}$$

Based on the total energy requirement of the appliances in a coalition Co_w, i.e., $\sum_{n=1}^{n \in \mathcal{C}_w} a_n$, the micro-grids need to decide among themselves the minimum amount of energy, G_{min}, required to be generated, and the minimum price per unit energy, p_{min}, to optimize the overall revenue of the micro-grids. To provide the minimum energy requirement of each customer n, i.e., a_n, each micro-grid decides the minimum price per unit energy, p_{min}, with the cooperation of other micro-grids. Each micro-grid $m \in \mathcal{W}$ tries to sell the excess amount of generated energy with a higher price per unit energy to maximize its revenue. Hence, an optimal price, which is neither too high nor too low, needs to be chosen by each micro-grid, to maximize its profit.

To complete energy trading successfully, proper interaction among the central energy management unit (CEMU), the micro-grids, and the customers is needed. We divide the interactions into two stages — *initialization phase with cooperation* (IPC), and *finalization phase with non-cooperation* (FPN). In IPC, each micro-grid m exchanges information with the CEMU to decide G_{min}, and p_{min}. In FPN, each customer n in a coalition Co_w needs to decide the amount of energy to be requested to the micro-grid m, and the micro-grid $m \in \mathcal{W}$ needs to decide the price per unit energy p_m, where $p_m \geq p_{min}$. However, p_m also depends on the total energy required by a customer n, e_n, and the number of customers under micro-grid m, $|\mathcal{C}_m|$. If the amount of energy acquire for appliances, a_n, is higher, the excess energy for storage, $\mathcal{S}_w$, will be reduced.

The energy requested by each customer has to fulfill the constrains given in Equation (3). It is also to be noted that the price decided by a micro-grid is also dependent on the amount of requested energy. Thus, the main challenges faced to develop the approach that can capture the two stages decision making processes are as follows:

(i) Modeling the decision making processes, the interactions between the micro-grids and the CEMU, and the micro-grids and the customers in the network, subject to the constrains in Equation (3).

(ii) Developing an algorithm for the micro-grids to decide G_{min} and p_{min}, by having interaction with the CEMU.

(iii) Developing another algorithm for the micro-grids to decide the amount of energy to be generated, and the actual price per unit energy p_m.

(iv) Each customer n needs to decide the total amount of energy e_n based on the optimally decided amount of energy for storage x_n to maximize its storage satisfaction level.

Communication between the Customer and Micro-Grid: We assume that the communication networking model between the micro-grids and the customers is wireless mesh network (WMN). We use the IEEE 802.11b protocol for the communication between the micro-grids and the customers. In the *Initialization Phase* (IP), each customer sends a message with the information of minimum energy requirement for the appliances, as shown in Figure 2(a).

In the *Finalization Phase* (FP), each micro-grid replies with the decided price per unit energy to the customers. The reply message format is shown in Figure 2(b). After receiving the reply message from the micro-grids, each customer decides how much energy s/he needs to consume, including the required energy for storage, and sends a request packet again to the micro-grids. The message format is shown in Figure 2(c). This message exchange continues until the customer decides an optimal value of requested energy, and the micro-grid gets the optimal price per unit energy.

IV. PROPOSED MULTIPLE LEADER MULTIPLE FOLLOWER STACKELBERG GAME

A. *Game formulation*

To study the interactions between the micro-grids and the customers, as mentioned earlier, we use a multiple leader multiple follower Stackelberg game. This is a multi-stage and multi-level game, where a group of players, i.e., the followers, take decision based on the decision of the leaders, using a non-cooperative game, and the leaders make decision among themselves using a cooperative game. In this paper, we consider the micro-grids as the leaders, and the customers as the followers. Hence, in the *Initialization Phase*, the micro-grids need to decide the amount of minimum energy to be generated, G_{min}, and the minimum price per unit energy, p_{min}, using a cooperative game theoretic approach. In the *Finalization Phase*, the customers need to decide the amount of energy to be requested, e_n, including the optimum amount of energy for storage, x_n, and the micro-grids need to decide the price per unit energy, p_m, using a non-cooperative game theoretic approach. The overall game is defined by using the strategic form, $\Upsilon = \{(\mathcal{N} \cup \mathcal{M}), (X_n, A_n, E_n, \psi_n)_{n \in \mathcal{N}}, (G_m, P_m, \varphi_m, p_m, \phi_m)_{m \in \mathcal{M}}, G_{min}, p_{min}\}$

The components in the strategic form Υ are as follows:

(i) Each customer n acts as a follower in the game, and needs to decide the optimum energy demand e_n, based on the optimum price decided by the micro-grid.

(ii) The strategy of each customer n is to decide the total amount of energy e_n from the micro-grid, while satisfying the constraints given in the Equations (3) and (5).

(iii) Each customer n optimizes the amount of energy to be stored, while satisfying the constraint – $\mathcal{S} \geq \sum_{n=1}^{n \in \mathcal{N}} x_n$, where $\mathcal{S}$, which is broadcasted to the customers within a coalition by the CEMU, is the total amount of excess energy that can be acquired by the customers for stored energy. Mathematically,

$$\mathcal{S} = \sum_{w=1}^{\left\lfloor \frac{|\mathcal{M}|}{|\mathcal{W}|} \right\rfloor} \mathcal{S}_w \tag{6}$$

(iv) The utility function $\psi_n(\cdot)$ of a customer n is used to maximize the payoff value by capturing the benefit of the total consumed energy e_n.

(v) The utility function $\varphi_m(\cdot)$ of a micro-grid m is used to maximize the payoff value of micro-grid m using the information of total consumed energy from micro-grid m.

(vi) The price p_m denotes the price per unit energy decided by the micro-grid m.

(vii) The utility function ϕ_m of a micro-grid m captures the minimum profit by selling the energy to fulfill the minimum energy requirement by the customers $\mathcal{C}_w$ in a coalition Co_w.

(viii) The energy G_{min} denotes the minimum energy needed to be generated by each micro-grid m.

The game formulation of the *Initialization* and the *Finalization Phases* of the multi leader multi follower Stackelberg game are discussed in Sections IV-A1, and IV-A2, respectively.

1) Game formulation for the Initialization Phase:

a) *Utility function of a micro-grid for Initialization Phase*: In the Initialization Phase, each micro-grid m, that acts as a leader, decides G_{min} and P_{min}, based on the minimum amount of requested energy by the customers, i.e., a_n; $\forall n \in \mathcal{C}_w$. The vector showing the amount of energy A_n, requested by each customer n, is the maximum expected energy vector to be needed for the appliances, and is forecasted on a day-ahead basis. Here, a_n^t, g_n^t, p_{min}^t are the minimum expected energy of a customer n for time slot t, the minimum energy to be generated for time slot t, and the minimum price per unit energy for time slot t, respectively.

Initially, in a coalition Co_w, each customer n calculates its expected amount of energy vector, A_n, and broadcasts to the micro-grids, $\mathcal{W}$. The micro-grid $m \in \mathcal{W}$ decides to generate g_m amount of energy to maximize its utility function $\phi_m(g_m, \mathbf{g}_{-m})$, while the price per unit energy p_m would be fixed for all the micro-grids in a coalition. Mathematically,

$$\arg\max_{g_m} \phi_m(g_m, \mathbf{g}_{-m}), \qquad \forall m \in \mathcal{W} \tag{7}$$

where $\mathbf{g}_{-m} = \{g_1, g_2, \cdots, g_{m-1}, g_{m+1}, \cdots, g_{|\mathcal{W}|}\}$. Equation (7) must satisfy the constraint given in Equation (3). Hence, the properties that the utility function must satisfy are as follows:

(i) The utility function of a micro-grid m, ϕ_m, is considered as a non-decreasing function. With the increase in energy demand, the total revenue of a micro-grid m increases. Mathematically,

$$\frac{\delta \phi_m(g_m, \mathbf{g}_{-m})}{\delta g_m} \geq 0, \qquad \forall m \in \mathcal{W} \text{ and } \forall n \in \mathcal{C}_w \tag{8}$$

(ii) If the total generated energy by a micro-grid m equals the total requested energy by a group of customers, i.e., $\sum_{n=1}^{n \in \mathcal{C}_w} a_n$, the utility function is considered to be in the marginal position. In this situation, the utility function of the micro-grids are considered to be non-increasing function. Mathematically,

$$\frac{\delta^2 \phi_m(g_m, \mathbf{g}_{-m})}{\delta {g_m}^2} < 0, \qquad \forall m \in \mathcal{W} \tag{9}$$

(iii) With the increase in the total amount of energy demand by the customers, $\sum_n a_n$, the payoff of the utility function ϕ_m increases. Mathematically,

$$\frac{\delta \phi_m(g_m, \mathbf{g}_{-m})}{\delta a_n} > 0, \qquad \forall m \in \mathcal{W}, \text{ and } \forall n \in \mathcal{C}_w \tag{10}$$

(iv) With a fixed amount of energy request, i.e., $\sum_n a_n$, if the price per unit energy p increases, the payoff of the utility function ϕ_m also increases. Mathematically,

$$\frac{\delta \phi_m(g_m, \mathbf{g}_{-m})}{\delta p} > 0, \qquad \forall m \in \mathcal{M} \tag{11}$$

The utility function ϕ_m denotes the maximum profit of micro-grid m that it can have by selling the minimum amount of energy. Therefore, the utility function ϕ_m becomes,

$$\phi_m(g_m, \mathbf{g}_{-m}) = p g_m - c_m g_m \tag{12}$$

where, c_m is the generation cost per unit energy for micro-grid

m, g_m is the generated energy by the micro-grid m. The total energy that needs to be generated by the micro-grids $\mathcal{W}$ in a coalition, $\mathcal{G}_{\mathcal{W}}$, is defined as,

$$\mathcal{G}_{\mathcal{W}} = \sum_{m=1}^{m \in \mathcal{W}} g_m \tag{13}$$

b) *Existence of Generalized Nash Equilibrium for the Initialization Phase*: In any optimization approach, there should be an optimal or Pareto-optimal solution. Therefore, we need to investigate the existence of generalized Nash equilibrium for the Initialization Phase. In this Phase, we find out the equilibrium point under the assumptions — in a coalition, (i) each micro-grid has the same generation cost per unit energy, c, and (ii) p_{min} would be fixed for all the micro-grids.

Definition 1. *While the total demand of energy for all the customers is fixed, with the increase in supply of the total amount of energy, the price per unit energy reduces. So, the price function varies inversely with the demand function. We formulate an inverse demand function $\mathcal{P}(\mathcal{G}_{\mathcal{W}})$ as follows:*

$$\mathcal{P}(\mathcal{G}_{\mathcal{W}}) = A - \mathcal{G}_{\mathcal{W}} \tag{14}$$

where A is a constant, and $\mathcal{G}_{\mathcal{W}}$ is the total generated energy by $\mathcal{W}$ micro-grids in the coalition Co_w. $\mathcal{G}_{\mathcal{W}}$ must satisfy the condition given in Equation (3).

Theorem 1. *If the generation cost per unit energy for each micro-grid is the same, the amount of energy to be generated by each micro-grid m, i.e., g_m, will be same, i.e., a generalize Nash equilibrium (GNE) point, if and only if the following inequality holds,*

$$\phi_m({g_m}^*, \boldsymbol{g}^*_{-m}) \geq \phi_m(g_m, \boldsymbol{g}^*_{-m}) \tag{15}$$

Proof: For the micro-grids m, the generation cost per unit energy, c_m, remains the same. The optimal energy supply of the the micro-grid m, i.e., the best response of micro-grid m, is defined as follows:

$$g^*_m(c_m) = \arg\max_{g_m}((A - \mathcal{G}_{\mathcal{W}}) - c_m)g_m \tag{16}$$

We rewrite the function by replacing c_m by c, where c is a constant.

$$\text{Therefore,} \quad g^*_m(c) = \arg\max_{g_m}((A - \mathcal{G}_{\mathcal{W}}) - c)g_m \tag{17}$$

$$\text{Hence,} \quad g^*_1(c) = \arg\max_{g_1}\left[\left(A - g_1 - g^*_2 - \sum_{m=3}^{m \in \mathcal{W}} g^*_m\right) - c\right]g_1 \tag{18}$$

$$\text{Similarly,} \quad g^*_2(c) = \arg\max_{g_2}\left[\left(A - g^*_1 - g_2 - \sum_{m=3}^{m \in \mathcal{W}} g^*_m\right) - c\right]g_2 \tag{19}$$

The optimal value of g_1, i.e., g^*_1, can be obtained from the necessary condition, as follows:

$$\left.\frac{\delta g_1(c)}{\delta g_1}\right|_{g_1 = g^*_1} = 0; \Rightarrow g^*_1 = \frac{A - g^*_2 - \sum_{m=3}^{m \in \mathcal{W}} g^*_m - c}{2} \tag{20}$$

Similarly, we get the optimum value of g_2 as follows:

$$g^*_2 = \frac{A - g^*_1 - \sum_{m=3}^{m \in \mathcal{W}} g^*_m - c}{2} \tag{21}$$

By solving Equations (20), and (21), we get,

$$g^*_1 = g^*_2 = A - c \tag{22}$$

From Equation (22), we infer that,

$$g^*_1 = g^*_2 = \cdots = g^*_m = \cdots = g^*_{|\mathcal{W}|}$$

Hence, within a coalition, each micro-grid m generates the same minimum amount of energy to satisfy the inequality for GNE, i.e., $\phi_m(g^*_m, \mathbf{g}^*_{-m}) \geq \phi_m(g_m, \mathbf{g}^*_{-m})$. ■

2) *Game Formulation for the Finalization Phase:* The interaction between the micro-grids and the customers in a coalition is evaluated using the second part of the multiple leader multiple follower Stackelberg game, where each micro-grid m acts as the leader, and the customers n act as the followers. Initially, each leader, i.e., micro-grid m, generates energy using renewable energy resources. The micro-grid m needs to generate energy using non-renewable energy resources, if the micro-grid does not satisfy the following inequality:

$$(G_{RE})_m \geq G_{min} \tag{23}$$

where $(G_{RE})_m$ is the amount of energy generated using renewable energy resources by micro-grid m. Therefore, we can define the amount of energy generated using non-renewable energy resources, $(G_{NE})_m$ by a micro-grid m is as follows:

$$(G_{NE})_{m_{min}} = \begin{cases} 0 & \text{if } (G_{RE})_m \geq G_{min} \\ G_{min} - (G_{RE})_m & \text{if } (G_{RE})_m < G_{min} \end{cases} \tag{24}$$

a) *Utility function of a customer*: For each customer $n \in \mathcal{C}_w$ in the coalition Co_w, we formulate the utility function $\psi_n(\cdot)$ to introduce the amount of energy requested to fulfill the requirement of the customers. In the utility function ψ_n, the maximum energy storage capacity of the customer n is denoted by $(E_{max})_n$, the stored energy of a customer n is denoted by $(E_{res})_n$, the total amount of energy requested by the customer n is denoted by e_n, and $\mathbf{e}_{-n}$ denotes the total amount of energy requested by the other customers in the coalition, except customer n, i.e., $\mathbf{e}_{-n} = \{e_1, e_2, e_3, \cdots, e_{n-1}, e_{n+1}, \cdots, e_{|\mathcal{C}_w|}\}$, needed to be predicted by customer n, where $|\mathcal{C}_w|$ is the number of customers in a coalition Co_w having the micro-grids $\mathcal{W} \subseteq \mathcal{M}$. Each customer n intends to increase its residual energy, $(E_{res})_n$, as that can be used by her/him at the on-peak hour of the day, and also in a blackout or islanding situation. So, having a fixed amount of maximum energy storage capacity, $(E_{max})_n$, the customer n requests higher e_n due to higher amount of energy needed for storage x_n. The amount of energy requested for storage will be affected by the decided price per unit energy, p_m, by micro-grid m. Thus the property of the utility function $\psi_n(\cdot)$ of a customer $n \in \mathcal{C}_w$ must satisfy the following conditions,

(i) The utility function ψ_n of the customer n is considered as a non-decreasing function, as each customer wants to acquire more amount of energy e_n to maximize its residual energy, $(E_{res})_n$. Mathematically,

$$\frac{\delta \psi_n(e_n, \mathbf{e}_{-n}, (E_{max})_n, (E_{res})_n, p_m)}{\delta e_n} \geq 0 \tag{25}$$

(ii) The limiting value of the utility function ψ_n of a customer n is considered to be a non-increasing function, as the residual energy $(E_{res})_n$ increases the amount of requested energy e_n. Mathematically,

$$\frac{\delta^2 \psi_n(e_n, \mathbf{e}_{-n}, (E_{max})_n, (E_{res})_n, p_m)}{\delta {e_n}^2} < 0 \tag{26}$$

(iii) If the amount of maximum energy storage capacity $(E_{max})_n$ is higher, the energy requirement of the customer n will be higher. So, the utility function ψ_n varies proportionally with $(E_{max})_n$. Mathematically,

$$\frac{\delta\psi_n(e_n, \mathbf{e}_{-n}, (E_{max})_n, (E_{res})_n, p_m)}{\delta(E_{max})_n} > 0 \tag{27}$$

(iv) If the amount of stored energy $(E_{res})_n$ decreases, the the energy requirement of the customer n increases. The utility function ψ_n has an inversely-proportional relationship with the amount of residual energy $(E_{res})_n$. Mathematically,

$$\frac{\delta\psi_n(e_n, \mathbf{e}_{-n}, (E_{max})_n, (E_{res})_n, p_m)}{\delta(E_{res})_n} < 0 \tag{28}$$

(v) The amount of requested energy, e_n, is affected by the price per unit energy, p_m, decided by the micro-grid m. With the higher value of price, the payoff of the utility function ψ_n of a customer n decreases. Mathematically,

$$\frac{\delta\psi_n(e_n, \mathbf{e}_{-n}, (E_{max})_n, (E_{res})_n, p_m)}{\delta p_m} < 0 \tag{29}$$

Therefore, the utility function ψ_n is formulated as follows:

$$\psi_n(\cdot) = (E_{max})_n e_n - \frac{1}{2}\alpha\frac{(E_{res})_n}{(E_{max})_n}{e_n}^2 - \beta\frac{p_m}{p_{min}}\mathcal{S}_w e_n \tag{30}$$

We consider that the transmission channel is ideal in nature, i.e., the resistance of the transmission channel is considered to be zero. Therefore, the transmission losses due to energy transfer is zero. Additionally, we consider that the energy transmission limit is taken care of by the electrical circuitry, i.e., the transformers. Hence, $\psi_n(e_n, \mathbf{e}_{-n}, (E_{max})_n, (E_{res})_n, p_m)$ must satisfy the following constrains,
(1) e_n is defined in Equation (2).
(2) The amount of requested energy for the appliances a_n by the customer n satisfies:

$$a_n \in \left[0, \sum_{m=1}^{m\in\mathcal{W}} g_m - \sum_{q=1,q\neq n}^{q\in\mathcal{C}_w} a_q\right] \tag{31}$$

(3) The amount of requested energy for the storage x_n by the customer n satisfies:

$$x_n \in \left[0, \sum_{m=1}^{m\in\mathcal{W}} g_m - \sum_{r=1}^{r\in\mathcal{C}_w} a_r - \sum_{q=1,q\neq n}^{q\in\mathcal{C}_w} x_q\right] \text{and} \sum_{n}^{n\in\mathcal{C}_w} x_n \leq \mathcal{S}_w \tag{32}$$

(4) α and β are constants, and have a fixed value within a coalition. These constants satisfy the following inequality,

$$\alpha, \beta > 0 \tag{33}$$

b) *Utility function of a micro-grid*: Each micro-grid $m \in \mathcal{W}$ gets a revenue of $p_m e_n$ by selling e_n amount of energy with p_m price per unit energy. Mathematically,

$$\varphi_m(e_n(p_m), p_m) = p_m \sum_n e_n \tag{34}$$

where p_m is the fixed price per unit energy for micro-grid m. However, each micro-grid knows that if the value of p_m is lower, the amount of energy requested by the customers is higher, and vice-versa, in either case it may get less revenue. So, the micro-grid m needs to choose an optimize value of p_m to maximize its revenue. Mathematically,

$$\arg\max \varphi_m(e_n(p_m), p_m) = \max_{p_m} \sum_m \sum_n p_m e_n \tag{35}$$

where $m \in \mathcal{W}$, $\mathcal{W} \subseteq \mathcal{M}$, and $p_m \geq p_{min}$.

The requested energy e_n of each customer n is not only dependent on the price per unit energy decided by the micro-grid, and the amount of required energy to fulfill its maximum storage capacity, i.e., $((E_{max})_n - (E_{res})_n)$, but also the requested energy by the other customers. Therefore, this scenario leads to a non-cooperative game that deals with sharing a common product having a fixed constraint for all. We will prove in Subsection IV-A2 that there exists generalized Nash equilibrium (GNE).

Definition 2. *The set of strategies* $(\{e_n^*\}_{n\in\mathcal{N}}, \{p_m^*\}_{m\in\mathcal{M}})$ *are considered as the generalized Nash equilibrium solutions, if those satisfy the following inequalities:*

$$\psi_n(e_n^*, \boldsymbol{e}_{-n}^*, \cdot, p_m^*) \geq \psi_n(e_n, \boldsymbol{e}_{-n}^*, \cdot, p_m^*) \textit{ and} \tag{36}$$

$$\varphi_m(e_n^*(p_m^*), p_m^*) \geq \varphi_m(e_n^*(p_m), p_m) \tag{37}$$

where e_n^* *is the optimum energy requested by the customer* n*, and* p_m^* *is the optimum price per unit energy decided by the micro-grid* m*. Each customer* n *cannot maximize the payoff of the utility function* ψ_n *by changing the value of* e_n *from the value of* e_n^**. Similarly, each micro-grid* m *cannot maximize the payoff of the utility function* φ_m *by choosing a higher price* p_m *than the price* p_m^**.*

c) *Existence of Generalized Nash Equilibrium for the Finalization Phase*: In this section, we determines the existence of GNE by showing that it satisfies the properties of *variation inequality* (VI), as it is used to get the optimum convex solution under some constraints of inequality.

Theorem 2. *Given a fixed price* p_m *by the micro-grid* m*, there exists a GNE, as there exists a variational equilibrium for the utility function* $\psi_n(e_n^*, \boldsymbol{e}_{-n}^*, (E_{max})_n, (E_{res})_n, p_m^*)$*.*

Proof: In the Finalization Phase, the utility function $\psi_n(\cdot)$ needs to be maximized. The utility function $\psi_{k,k\neq n}(\cdot)$, where $k \in \mathcal{C}_w$, also needs to be maximized.

$$\psi_{k,k\neq n}(\cdot) = (E_{max})_k e_k - \frac{1}{2}\alpha\frac{(E_{res})_k}{(E_{max})_k}{e_k}^2 - \beta\frac{p_m}{p_{min}}\mathcal{S}_w e_k \tag{38}$$

From Equations (30) and (38), we get,

$$\psi(\cdot) = \sum_n (E_{max})_n e_n - \frac{1}{2}\alpha\sum_n \frac{(E_{res})_n}{(E_{max})_n}{e_n}^2 - \beta\frac{p_m}{p_{min}}\mathcal{S}_w \sum_n e_n \tag{39}$$

Using the method of Lagrangian multiplier, the *Karush-Kuhn-Tucker* (KKT) condition of the customer n for the GNE problem becomes:

$$\nabla_{e_n}\psi_n(\cdot) - \nabla_{e_n}\left(\sum_n x_n - \mathcal{S}_w\right)\mu_n = 0, \left(\sum_n x_n - \mathcal{S}_w\right)\mu_n = 0 \tag{40}$$

$$\text{and} \qquad \mu_n \geq 0 \tag{41}$$

where μ_n is the Lagrangian multiplier for the customer n.

By using the property of variational inequality (VI), we get VI$(\mathbf{B}, \mathbf{X})$ as the solution of the variational equilibrium, where $\mathbf{X}$ is the set of optimum points for x, and $\mathbf{B} = \nabla_{e_n}\psi_n(\cdot)$. We get the Jacobian matrix of $\mathbf{B}$ as follows,

$$\mathbf{J_B} = \nabla_\mathbf{e}\mathbf{B} = \begin{bmatrix} (E_{max})_1 - \alpha\frac{(E_{res})_1}{(E_{max})_1}e_1 - \beta\frac{p_m}{p_{min}}\mathcal{S}_w \\ \vdots \\ (E_{max})_{|\mathcal{C}_w|} - \alpha\frac{(E_{res})_{|\mathcal{C}_w|}}{(E_{max})_{|\mathcal{C}_w|}}e_{|\mathcal{C}_w|} - \beta\frac{p_m}{p_{min}}\mathcal{S}_w \end{bmatrix} \tag{42}$$

The Hessian matrix of $\mathbf{B}$ is the Jacobian matrix of $\nabla_{\mathbf{e}}\mathbf{B}$. Mathematically,

$$\mathbf{H_B} = \mathbf{J}(\nabla_{\mathbf{e}}\mathbf{B}) = \begin{bmatrix} -\alpha\frac{(E_{res})_1}{(E_{max})_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\alpha\frac{(E_{res})_{|\mathcal{C}_w|}}{(E_{max})_{|\mathcal{C}_w|}} \end{bmatrix} \tag{43}$$

As the Hessian matrix $\mathbf{H_B}$ is a diagonal matrix, we infer that vector $\mathbf{e}$ has a unique solution, where $\mathbf{e} = \{e_1, \cdots, e_{|\mathcal{C}_w|}\}$, and the variational equilibrium exists. Therefore, for a fixed price, there exists a GNE. ■

B. Why Stackelberg game?

In HoMeS, the micro-grids and the customers perform a sequential competition within themselves. Initially, the micro-grids decide the minimum price per unit energy, p_{min}, based on the minimum amount of energy to consumed by each customer n, a_n. On the other hand, the customers decide the actual amount of energy to be consumed, e_n, including the amount of energy required for storage, x_n, based on price decided by the micro-grids, p_m with initial condition $p_m|_{t=0}= p_{min}$. Sequentially, the micro-grids modify the price per unit energy, p_m, based on the value of e_n. This process continues till the equilibrium point is reached. Hence, for modeling the proposed scheme, HoMeS, we use multiple leader multiple follower Stackelberg game.

C. Proposed Solution Approach

From Section IV-A, we get that GNE exists for the multiple leader multiple follower Stackelberg game theoretic approach used in HoMeS. In this section, we compute the optimum solutions of the unknown variables.

a) *Solution approach for the Initialization Phase*: In the Initialization Phase, the minimum amount of energy to be generated by each micro-grid m, G_{min}, and the minimum price per unit energy, i.e., p_{min}, are decided, where the generation cost per unit energy c is fixed for the micro-grids $m \in \mathcal{M}$.

Definition 3. *In a coalition, the price per unit energy, p_{min} is the same as the generation cost per unit energy c. Mathematically,*

$$p_{min} = c\,, \qquad c > 0 \tag{44}$$

If for a micro-grid m, the price per unit energy, p_m, is the same as the generation cost per unit energy c, i.e., p_{min}, then the profit of the micro-grid m equals zero.

Lemma 1. *In a coalition, each micro-grid needs to generate the same minimum amount of energy to fulfill customers' energy demand.*

Proof: From Theorem 1 and Equation (13), we get,

$$g_1^* = \frac{g_2^* + g_3^* + g_4^* + \cdots + g_{|\mathcal{W}|}^*}{|\mathcal{W}| - 1} \tag{45}$$

We rewrite Equation (45) as follows:

$$g_1^* = \frac{g_1^* + g_2^* + g_3^* + \cdots + g_{|\mathcal{W}|}^*}{|\mathcal{W}|} = \frac{\sum\limits_{m=1}^{m\in\mathcal{W}} g_m^*}{|\mathcal{W}|} \tag{46}$$

Therefore, the minimum energy to be generated by each micro-grid m is same, as given in Equation (46). ■

b) *Solution approach for the Finalization Phase*: In this section, the value of the optimum amount of energy requested by each customer n, e_n^*, given the fixed price per unit energy p_m, and the value of optimum price, p_m^*, for the given optimum amount of energy e_n^*, is computed.

For each customer n, solving the KKT condition for the GNE problem defined in Equation (40), we get:

$$(E_{max})_n - \alpha\frac{(E_{res})_n}{(E_{max})_n}e_n - \beta\frac{p_m}{p_{min}}\mathcal{S}_w - \mu_n = 0 \tag{47}$$

From Equation (41), we get $\mu_n \geq 0$. Therefore,

$$(E_{max})_n - \alpha\frac{(E_{res})_n}{(E_{max})_n}e_n - \beta\frac{p_m}{p_{min}}\mathcal{S}_w \geq 0 \tag{48}$$

Solving Equation (48), we get,

$$e_n \leq \frac{(E_{max})_n}{\alpha(E_{res})_n}\left[(E_{max})_n - \beta\frac{p_m}{p_{min}}\mathcal{S}_w\right] \tag{49}$$

$$p_m \leq \frac{p_{min}}{\beta\mathcal{S}_w}\left[(E_{max})_n - \alpha\frac{(E_{res})_n}{(E_{max})_n}e_n\right] \tag{50}$$

So, the optimal values of e_n and p_m are as follows:

$$e_n^* = \frac{(E_{max})_n}{\alpha(E_{res})_n}\left[(E_{max})_n - \beta\frac{p_m^*}{p_{min}}\mathcal{S}_w\right] \tag{51}$$

$$p_m^* = \frac{p_{min}}{\beta\mathcal{S}_w}\left[(E_{max})_n - \alpha\frac{(E_{res})_n}{(E_{max})_n}e_n^*\right] \tag{52}$$

$$\mu_n = (E_{max})_n - \alpha\frac{(E_{res})_n}{(E_{max})_n}e_n^* - \beta\frac{p_m^*}{p_{min}}\mathcal{S}_w \tag{53}$$

D. Proposed Algorithm

In order to reach the equilibrium in home energy management system, the micro-grids and the customers take their respective strategies, while incurring a marginal communication overhead. In this paper, we propose two different algorithms — the *initialization phase with cooperation* (IPC) algorithm, and the *finalization phase with non-cooperation* (FPN) algorithm. In the IPC algorithm, the customers provide their minimum energy consumption profile for appliances on a day-ahead basis. After getting the information, the micro-grids communicate within themselves, i.e., cooperate, to finalize the values of G_{min} and p_{min}. In the FPN algorithm, after getting the minimum price per unit energy decided by the micro-grids, the customers communicate with the corresponding micro-grids, and decide the amount of actual energy to be consumed, e_n. After getting the actual consumption profile of the customers, each micro-grid m decides the actual price per unit energy, p_m, on a real-time basis. The micro-grids again broadcast the price per unit energy, p_m, and the customers may change their strategies, i.e., the value of e_n. This iterative process is performed between the customers and the micro-grids until equilibrium is reached. After reaching the equilibrium point, the micro-grids broadcast the same price as in the previous iteration. Consequently, the amount of energy to be consumed by each customer gets fixed.

1) Initialization Phase with Cooperation Algorithm: Initially, each customer n broadcasts a vector A_n representing his/her energy consumption profile for the appliances. Based on that information, the micro-grids decide the amount of energy to be generated by each micro-grid to fulfill the minimum

Algorithm 1 IPC algorithm for each micro-grid

```
Input: $A_n$: Broadcast energy consumption vector for appliances of customer $n$
Outputs: $G_{min}$: The minimum energy to be generated by each micro-grid $m$
         $p_{min}$: The minimum price per unit energy decided by each micro-grid $m$

   while $\sum_{m=1}^{m \in \mathcal{W}} g_m < \sum_{n=1}^{n \in \mathcal{C}_w} a_n$
     if $\phi_m(g_m^*, \mathbf{g}_{-m}^* \ngeq \phi_m(g_m, \mathbf{g}_{-m}^*)$
1.       Optimized value of $g_m$, i.e., $g_m^*$ is found
   else
2.       Evaluate the amount of energy to be generated, $g_m^{modified}$
3.       Decide the minimum energy to be generated, $g_m = g_m^{modified}$
     end if
   end while
4. Each micro-grid $m$ decides the minimum price per unit energy, $p_{min}$
5. Calculate minimum profit = $(p_{min} - c)g_m$
   while $(p_{min} - c) < 0$
6.     Decide higher value of $p_{min}$, $p_m^{modified}$
7.     The minimum price per unit energy, $p_{min} = p_m^{modified}$, is computed
   end while
```

Algorithm 2 FPN algorithm for a customer

```
Inputs: $p_m^*$: The optimum price per unit energy
        $\mathcal{S}_w$: Total energy for storage
Output: $e_n^*$: Amount of energy to be served

1. Decide the amount of energy to be requested, $e_n^*$, by customer $n$
   while $\psi_n(e_n^*, \mathbf{e}_{-n}^*, \cdot, p_m^*) \ngeq \psi_n(e_n, \mathbf{e}_{-n}^*, \cdot, p_m^*)$
2.     $e_n = e_n^*$
3.     Evaluate the modified value of energy to be requested, $e_n^{modified}$
4.     $e_n^* = e_n^{modified}$
   end while
```

requirement of the customers, as discussed in Algorithm 1. The micro-grids also make an agreement within themselves to decide the minimum price per unit energy.

2) Finalization Phase with Non-cooperation Algorithm: In the Finalization Phase, the customers and the micro-grids execute two different algorithms — Algorithms 2 and 3, respectively. The customers decide the amount of energy to be requested, including the amount of energy for storage, based on the optimum price decided by the micro-grids. The micro-grids need to decide the actual price per unit energy, p_m, where $p_m \geq (p_m)_{min}$.

V. PERFORMANCE EVALUATION

A. *Simulation Settings*

For performance evolution, we considered randomly generated values for the micro-grids and the customers, as shown in Table I, on a MATLAB simulation platform. The micro-grids form a coalition, based on the total energy requirement of the customers, the generation capacity of the micro-grids, and the area covered by the coalition, as discussed in [9].

Algorithm 3 FPN algorithm for a micro-grid

```
Input: $e_n^*$: Amount of energy to be served
Output: $p_m^*$: The optimum price per unit energy

1. Decide the price per unit energy, $p_m^*$, by micro-grid $m$
   while $\varphi_m(e_n^*(p_m^*), p_m^*) \ngeq \varphi_m(e_n^*(p_m), p_m)$
2.     $p_m = p_m^*$
3.     Evaluate the modified value of price per unit energy, $p_m^{modified}$
4.     $p_m^* = p_m^{modified}$
   end while
```

TABLE I: Simulation Parameters

Parameter	Value
Simulation area	20×20 km^2
Number of micro-grids	10
Number of Customers	1000
Minimum requested energy for appliances	90 MWh
Maximum requested energy for appliances	100 MWh
Customer's minimum storage capacity	35 MWh
Customer's maximum storage capacity	65 MWh
Customer's minimum residual stored energy	20 KWh
Minimum renewable energy generated	500 MWh
Maximum renewable energy generated	650 MWh
Generation cost	10-20 USD/MWh

B. *Benchmarks*

The performance of the proposed scheme, HoMeS, is evaluated by comparing it with other energy management policies, such as the economics of electric vehicle charging (E2VC) [24] approach, and the price taking user (PTU) [25] approach.

We refer to these different energy management policies as HoMeS, E2VC, and PTU, through the rest of the paper. Tushar *et al.* [24] proposed a game theoretic approach with storage. Samadi *et al.* [25] proposed a home energy management system without storage. Though E2VC [24] has been used for energy management system of the PHEVs, its authors did not consider any mobility model such as random way-point model or Gauss Markov mobility model for the PHEVs. Thus, we can improve the efficiency in the home energy management system by using our proposed approach, HoMeS, over E2VC and PTU.

C. *Performance Metrics*

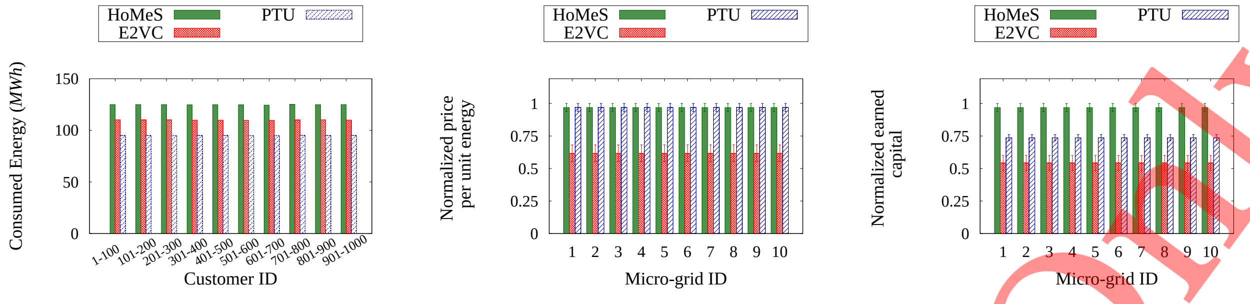
(i) *Real-time pricing policy for storage*: The price is decided by the micro-grids based on the real-time communication with the customers.

(ii) *Utility of the customers*: Each customer tries to maximize the payoff of its utility function that symbolizes the energy consumption with optimal price. A customer tries to maximize its utility by maximizing its energy consumption, while satisfying the inequality given in Equation (36).

(iii) *Consumed energy by the customers*: The amount of energy to be consumed for the appliances is decided on a day-ahead basis, whereas the actual energy to be consumed by each customer is decided by the customers in real-time. So, effectively, the energy consumed by the customers is decided by real-time home energy management system, and the lower limit of the consumed energy is decided *a priori*.

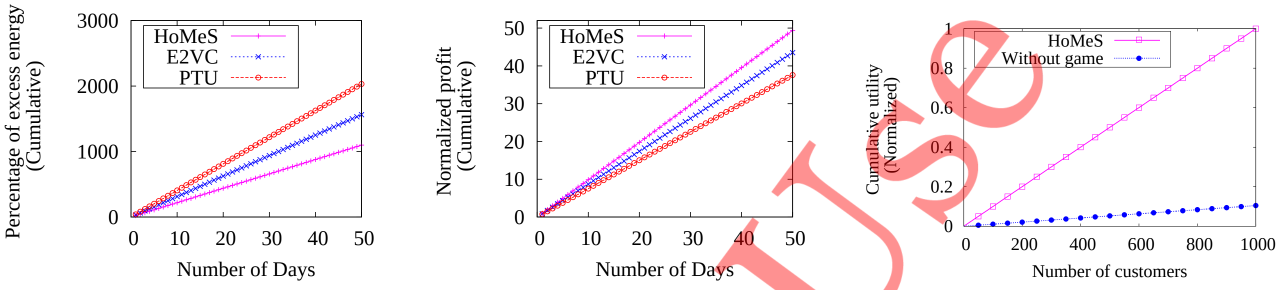
D. *Results and Discussions*

For the sake of simulation, we assume that each micro-grid calculates the real-time supply and demand in every 10 seconds interval. In Figure 3(a), the comparison of consumed energy, e_n, by each customer n is shown, where the energy consumption for the appliances is same for HoMeS, E2VC, and PTU. The customer decide the energy to be requested for storage on a real-time basis. Figure 3(a) shows that the consumed energy our proposed method, HoMeS, is 30% and 55% higher than E2VC and PTU, respectively. Using E2VC,



(a) Energy consumption of the customers (b) Price decided by the micro-grids (c) Earned Capital of the micro-grids

Fig. 3: Comparison of Energy Consumption of Customers, and Price and Profit of Micro-grids



(a) Excess energy (b) Profit over days (c) Utility of customers

Fig. 4: Comparison of Excess Energy and Profit over Days and Utility of Customers

the PHEVs consume energy for storage devices at the PHEV-end, whereas using PTU, the customers are not equipped with any storage facility. On the other hand, using HoMeS, the customers can fulfill their daily energy requirement for the appliances. Additionally, using HoMeS, the customers can also utilize the storage facility at his/her-end while consuming higher amount of energy. Therefore, the energy generated by the micro-grids is more adequately utilized using HoMeS, than using the other approaches — E2VC and PTU.

In Figure 3(b), the comparison of price per unit energy, p_m, is shown. p_m using E2VC is lower than using HoMeS and PTU. However, the capital earned by selling the generated energy by the micro-grids is much higher using HoMeS, than using other approaches — E2VC and PTU, as shown in Figure 3(c). The revenue of each micro-grid depends on the price per unit energy and the supplied amount of energy, i.e., $p_m \sum e_n$. Using HoMeS, the supplied amount of energy is much higher than using E2VC anf PTU, as shown in Figure 3(a). Therefore, each micro-grid, using HoMeS, earns higher than using E2VC and PTU.

Figure 4(a) shows that the percentage of excess energy, generated by the micro-grids, is also lower for HoMeS than E2VC and PTU. Therefore, Figure 4(a) re-establishes the fact that the energy generated by the micro-grids is more adequately utilized using HoMeS than using E2VC and PTU, as concluded from Figure 3(a).

Figure 4(b) shows that the overall profit of the micro-grids in a coalition is 15.39% and 30.79% higher using HoMeS, than using E2VC and PTU, respectively. In Figure 4(b), the cumulative profit of the micro-grids is shown. On the other hand, Figure 5 shows the profit of each micro-grid, individually. Therefore, each micro-grid, using HoMeS, gets higher profit than using E2VC and PTU, and the overall profit of the coalition formed by the micro-grids is also higher using HoMeS than using other approaches, i.e., E2VC and PTU. Figure 4(c) shows the utility of the customers, which combines the effect of utilization of energy generated by the micro-grids, energy consumption of the customers with optimum price, and the profit of the micro-grids, varies significantly using HoMeS, which uses the multiple leader multiple follower Stackelberg game theoretic approach, than using a different approach. Therefore, with the increase in the number of customers, the utility of the micro-grids is much higher using HoMeS than using any non-game theoretic approach.

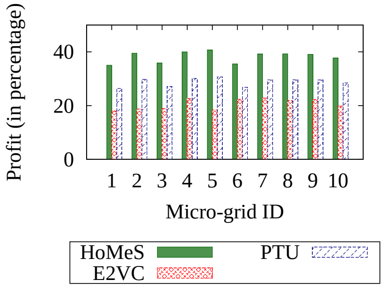


Fig. 5: Profit of micro-grids

VI. CONCLUSION

In this paper, we formulated a multiple leader multiple follower Stackelberg game theoretic approach, named HoMeS, to study the problem of distributed home energy management system with storage facilities. Using the proposed approach, we showed how distributed energy management system for the home appliances in the presence of storage can be done with the optimum value of the energy requested by the customers, while considering the overall energy demand in the system. On the other hand, the profit of the micro-grids is also ensured, while the optimum price decided by each micro-grid is less compared to that using the traditional energy distribution mechanism. The simulation results show that the proposed approach yields improved results.

Future extension of this work includes understanding how the energy distribution can be improved by exchanging less number of messages, so that the delay in energy supply can be reduced, and the service provided by the micro-grids to the customers can be improved, thereby improving the utilization of the micro-grids.

REFERENCES

[1] M. E. Kantarci and H. T. Mouftah, "Smart Grid Forensic Science: Applications, Challenges, and Open Issues," *IEEE Comm. Mag.*, vol. 51, no. 1, pp. 68–74, 2013.

[2] A. H. M. Rad, V. W. Wong, J. Jatskevich, and R. Schober, "Optimal and Autonomous Incentive-based Energy Consumption Scheduling Algorithm for Smart Grid," in *IEEE Inn. Smart Grid Tech.*, 2010, pp. 1–6.

[3] S. Misra, S. Bera, and M. S. Obaidat, "Economics of Customer's Decisions in Smart Grid," *IET Net.*, vol. 3, no. 1, pp. 1–7, Apr. 2014.

[4] D. Niyato, L. Xiao, and P. Wang, "Machine-to-Machine Communications for Home Energy Management System in Smart Grid," *IEEE Comm. Mag.*, vol. 49, no. 4, pp. 53–59, Apr. 2011.

[5] E. Santacana, G. Rackliffe, L. Tang, and X. Feng, "Getting smart," *IEEE Power and Energy Mag.*, vol. 8, 2010.

[6] A. Ipakchi and F. Albuyeh, "Grid of the Future," *IEEE Power and Energy Mag.*, pp. 52–62, 2009.

[7] W. Saad, Z. Han, and H. Poor, "Coalitional Game Theory for Cooperative Micro-Grid Distribution Networks ," in *IEEE Int. Conf. on Comm. Wrkshp*, Kyoto, Japan, Jun. 2011, pp. 1–5.

[8] W. Saad, Z. Han, H. V. Poor, and T. Basar, "Game-Theoretic Methods for the Smart Grid: An Overview of Microgrid Systems, Demand-Side Management, and Smart Grid Communications," *IEEE Signal Processing Mag.*, vol. 29, no. 5, pp. 86–105, Sept. 2012.

[9] A. Mondal and S. Misra, "Dynamic Coalition Formation in a Smart Grid: A Game Theoretic Approach," in *Proc. of IEEE Int. Wrkshp on Smart Comm. Protocols and Algo. with ICC*, Jun. 2013, pp. 1067 – 1071.

[10] M. Such and C. Hill, "Battery energy storage and wind energy integrated into the Smart Grid," in *Proc. of IEEE PES on Innovative Smart Grid Techn.*, Washington, DC, Jan. 2012, pp. 1–4.

[11] S. Misra, A. Mondal, S. Banik, M. Khatua, S. Bera, and M. S. Obaidat, "Residential Energy Management in Smart Grid: A Markov Decision Process-Based Approach," in *IEEE Int. Conf. on Internet of Things*, Beijing, Chaina, Aug. 2013, pp. 1152–1157.

[12] V. Bakker, M. G. C. Bosman, A. Molderink, J. L. Hurink, and G. J. M. Smit, "Demand Side Load Management Using a Three Step Optimization Methodology," in *IEEE Int. Conf. on Smart Grid Comm.*, Oct. 2010, pp. 431–436.

[13] Y. Wang, W. Saad, Z. Han, H. Poor, and T. Basar, "A Game-Theoretic Approach to Energy Trading in the Smart Grid," *IEEE Trans. on Smart Grid*, vol. 5, no. 3, pp. 1439–1450, May 2014.

[14] S. Misra, S. Bera, and T. Ojha, "D2P: Distributed Dynamic Pricing Policy in Smart Grid for PHEVs Management," *IEEE Trans. on Parallel and Distributed Systems*, 2014, doi: 10.1109/TPDS.2014.2315195.

[15] A. Molderink, V. Bakker, M. G. C. Bosman, J. L. Hurink, and G. J. M. Smit, "Management and Control of Domestic Smart Grid Technology," *IEEE Trans. on Smart Grid*, vol. 1, no. 2, pp. 109–119, Aug. 2010.

[16] P. Samadi, H. M. Rad, V. W. S. Wong, and R. Schober, "Real-Time Pricing for Demand Response Based on Stochastic Approximation," *IEEE Trans. on Smart Grid*, vol. 5, no. 2, pp. 789–798, 2014.

[17] X. Fang, S. Misra, G. Xue, and D. Yang, "Smart Grid – The New and Improved Power Grid: A Survey ," *IEEE Comm. Surveys & Tutorials*, vol. 14, no. 4, pp. 944–980, Dec. 2011.

[18] M. E. Kantarci and H. T. Mouftah, "TOU-Aware Energy Management and Wireless Sensor Networks for Reducing Peak Load in Smart Grids," in *IEEE* 72^{nd} *Vehicular Tech. Conf. Fall*, Sept. 2010, pp. 1 – 5.

[19] A. Soroudi and T. Amraee, "Decision Making Under Uncertainty in Energy Systems: State of the Art," *Renewable and Sustainable Energy Reviews*, vol. 28, pp. 376–384, 2013.

[20] A. Soroudi, R. Caire, N. Hadjsaid, and M. Ehsan, "Probabilistic Dynamic Multi-Objective Model for Renewable and Non-Renewable Distributed Generation Planning," *IET Generation, Transmission & Distribution*, vol. 5, no. 11, pp. 1173–1182, Nov. 2011.

[21] A. Soroudi and M. Afrasiab, "Binary PSO-based Dynamic Multi-Objective Model for Distributed Generation Planning Under Uncertainty," *IET Renewable Power Gen.*, vol. 6, no. 2, pp. 67–78, Mar. 2012.

[22] A. Soroudi, M. Ehsan, R. Caire, and N. Hadjsaid, "Possibilistic Evaluation of Distributed Generations Impacts on Distribution Networks," *IEEE Trans. on Power Systems*, vol. 26, no. 4, pp. 2293–2301, Nov. 2011.

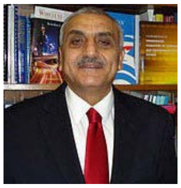
[23] A. Soroudi, "Possibilistic-Scenario Model for DG Impact Assessment on Distribution Networks in an Uncertain Environment," *IEEE Trans. on Power Systems*, vol. 27, no. 3, pp. 1283–1293, Aug. 2012.

[24] W. Tushar, W. Saad, H. V. Poor, and D. B. Smith, "Economics of Electric Vehicle Charging: A Game Theoretic Approach," *IEEE Trans. on Smart Grid*, vol. 3, no. 4, pp. 1767–1778, 2012.

[25] P. Samadi, H. M. Rad, R. Schober, and V. W. S. Wong, "Advanced Demand Side Management for the Future Smart Grid Using Mechanism Design," *IEEE Trans. on Smart Grid*, vol. 3, no. 3, pp. 1170–1180, 2012.

Ayan Mondal (S'13) is presently pursuing his Joint MS-PhD degree from School of Information Technology, Indian Institute of Technology Kharagpur, India. His current research interests include algorithm design for smart grid and wireless sensor networks. He received his B.Tech degree in Electronics and Communication Engineering from West Bengal University of Technology, India in 2012. He is a student member of IEEE and ACM.

Sudip Misra (SM'11) is an Associate Professor at the Indian Institute of Technology Kharagpur. He received his Ph.D. degree from Carleton University, Ottawa, Canada. Dr. Misra is the author of over 200 scholarly research papers. He has won several national and international awards including the IEEE ComSoc Asia Pacific Young Researcher Award during IEEE GLOBECOM 2012. Dr. Misra was also invited to deliver keynote/invited lectures in over 30 international conferences in USA, Canada, Europe, Asia and Africa.



Mohammad S. Obaidat (F'05) is an internationally well-known academic/researcher/scientist. He received his Ph.D. in Computer Engineering from Ohio State University. He currently a full Professor of Computer Science at Monmouth University,USA. Among his previous positions are Chair of Computer Science Department and Director of Graduate Program at Monmouth University. He served as advisor to President of Philadelphia University and as President, Senior VP, VP Membership and VP Conferences of SCS. He has received extensive research funding and has published over Thirty Five (35) books and 600 refereed technical articles. He is Editor-in-Chief of 3 scholarly journals and an editor of many other international journals. He received numerous awards. He is Fellow of IEEE and SCS.