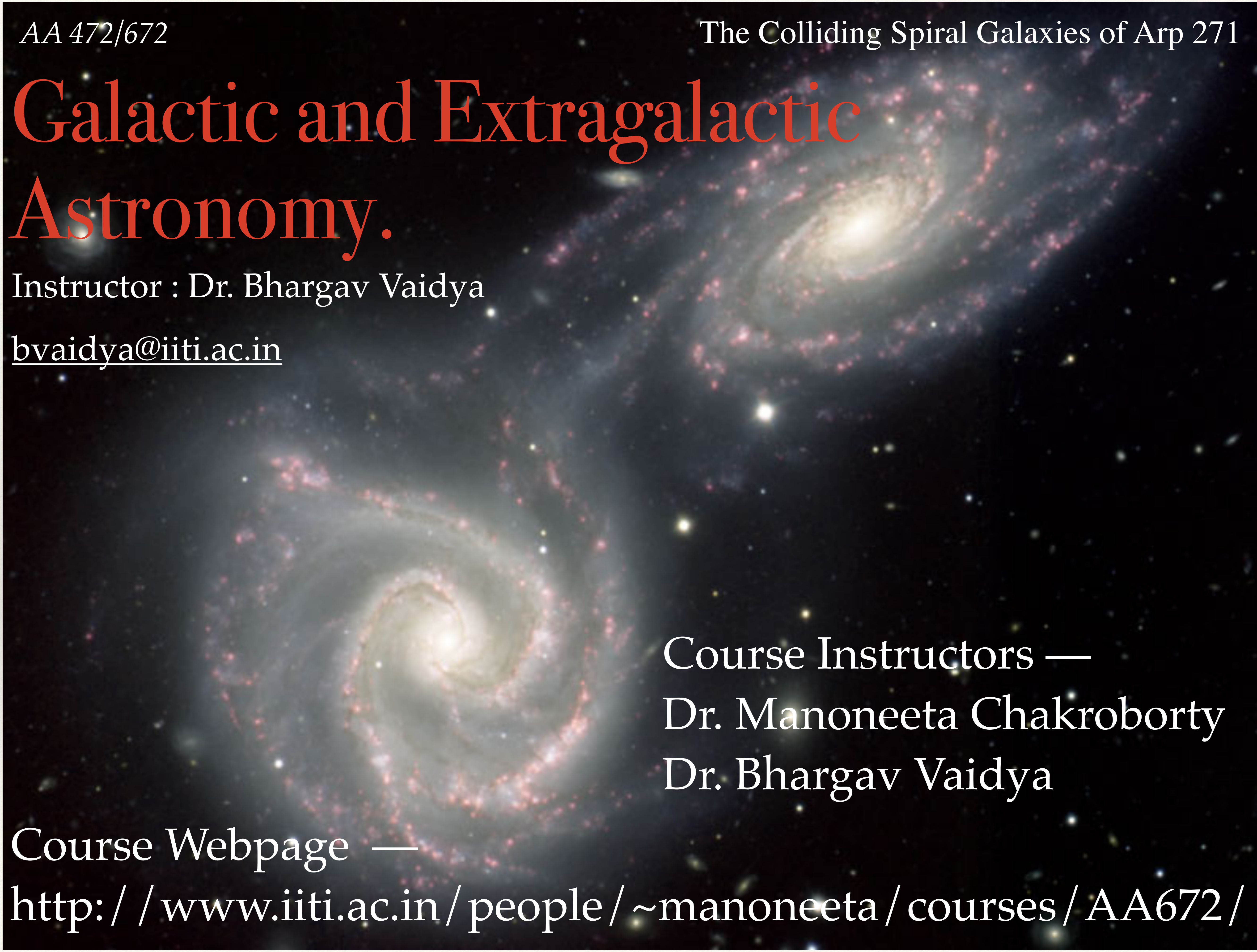
AA 472/672



The Colliding Spiral Galaxies of Arp 271

Galactic and Extragalactic Astronomy.

Instructor : Dr. Bhargav Vaidya

bvaidya@iiti.ac.in

Course Instructors —

Dr. Manoneeta Chakroborty

Dr. Bhargav Vaidya

Course Webpage —

http://www.iiti.ac.in/people/~manoneeta/courses/AA672/

Conservative Force.

- Total energy is conserved —> E = k.e + p.e. = constant.
- Work done by the force in moving the particle between two points is independent of the path taken.

$$\vec{F} \cdot d\vec{r} = -dV \rightarrow V = -\int F(r)dr; \vec{F} = -\nabla V$$

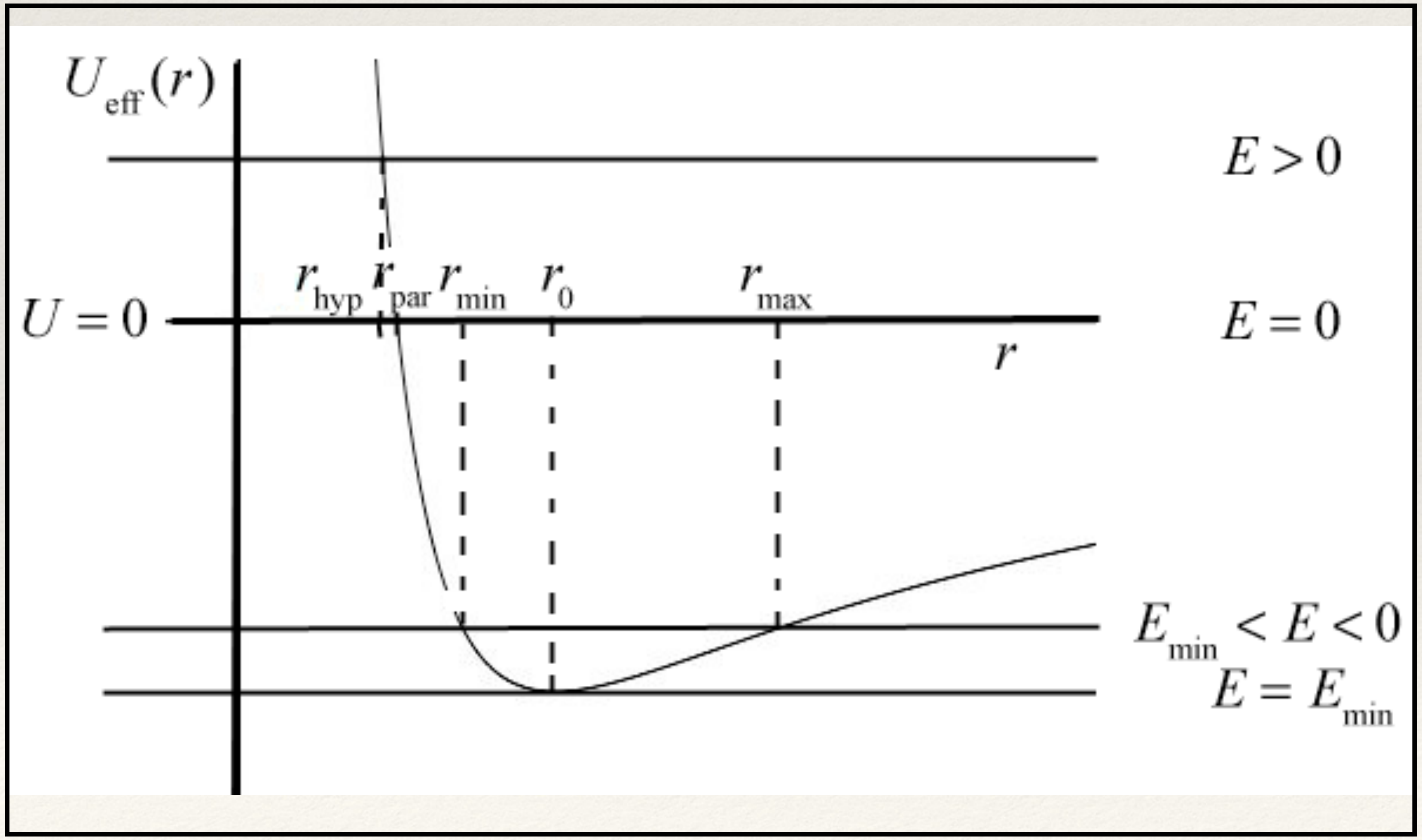
Total Energy

$$E = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \int F(r)dr$$

Effective Potential

- The effective potential < total energy.

$$E = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \int F(r)dr$$



$$E = \frac{1}{2}m\dot{r}^2 + V(r) + \frac{L^2}{2mr^2}$$

$$E = K_{\mathrm{eff}} + U_{\mathrm{eff}}$$

$$K_{\mathrm{eff}} = \frac{1}{2}m\dot{r}^2 > 0$$

$$U_{\mathrm{eff}} = V(r) + \frac{L^2}{2mr^2}$$

Orbits and its nature.

Circular Orbit

$$E = E_{\min} = U_{\text{eff}}(r_0) \rightarrow \left.\frac{dU_{\text{eff}}}{dr}\right|_{r=r_0} = 0$$

Ecliptic Orbit

$$E_{\min} < E < 0 : K_{\text{eff}} = 0 \text{ at } r_{\min}, r_{\max}$$

Parabolic Orbit

$$E = 0 \rightarrow K_{\text{eff}}(r = r_{\text{par}}) = -U_{\text{eff}}(r = r_{\text{par}})$$

Hyperbolic Orbit

$$E > 0$$

Bound Orbits in Spherical potential

Equation of Motion : $$\frac{d^2u}{d\theta^2} + u = \frac{1}{L^2u^2}\left[\frac{dV}{dr}(1/u)\right] ; m = 1; L = \mathcal{C}$$

$$\left(\frac{du}{d\theta}\right)^2 + u^2 = \frac{2}{L^2}(E - V)$$

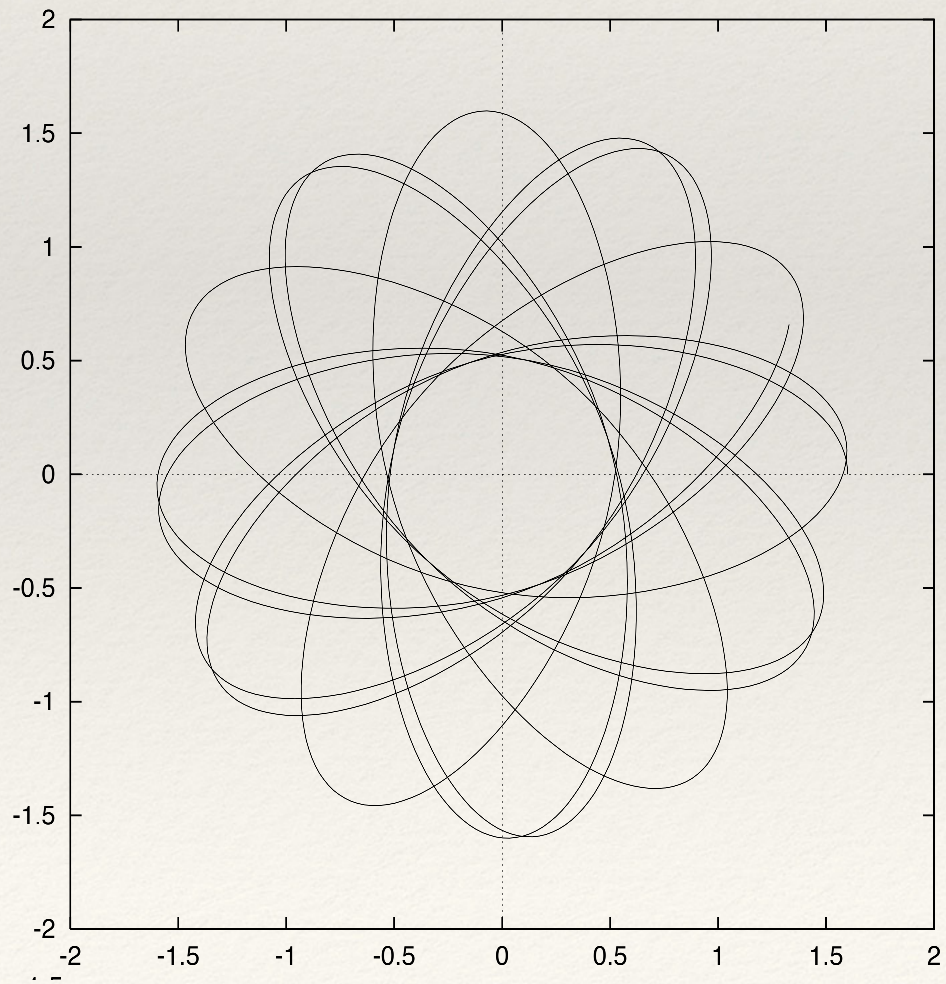
Two roots : Apocenter and Pericenter. They are equal for circular orbit and distance between them is a measure of eccentricity.

Radial and Azimuthal Period

Orbit will be closed if ratio between two time periods is a rational number.

$$T_\theta = \frac{2\pi}{\Delta\theta} T_r$$

Generally not true except for potential for a point mass and a homogenous sphere. … otherwise its a Rosette.



Galactic Dynamics.

- **Preliminaries**
  - Central Force: Definition, Properties, EoM and examples like motion of stars in spherically symmetric potential.
- **Galactic Dynamics**
  - *Potential Theory - spherical and flattened models , example : Milky Way Components and Potential.*
  - Motion of stars in axisymmetric potential
  - Two body system and relaxation Time.

Potential Theory in Galaxies.

- A simple description of the mass distribution and gravitational potential of a galaxy is to represent the stars as point masses.
- This is a good approximation for elliptical galaxies, since these usually contain very little gas.
- It is an adequate approximation for spiral/disk galaxies, because they usually contain significantly more stars than gas.

Newton's Spherical Shell Theorem.

- A body inside a spherical shell experiences no net gravitational force from that shell.
- A body outside a spherical shell experiences a gravitational force equal to the force of a mass point in the centre of the shell with the mass of the shell.
- *Exercise : Prove both statements.*

Potential Density Pair of a Galaxy.

To calculate the force F(x) on a particle of mass m at position r that is generated by the gravitational attraction of a distribution of mass ϱ(r′).

According to Newton's inverse-square law of gravitation, the force F(x) may be obtained by summing the small contributions

Differential Form

$$\text{Given} \quad \Phi(r) \to \rho(r)$$

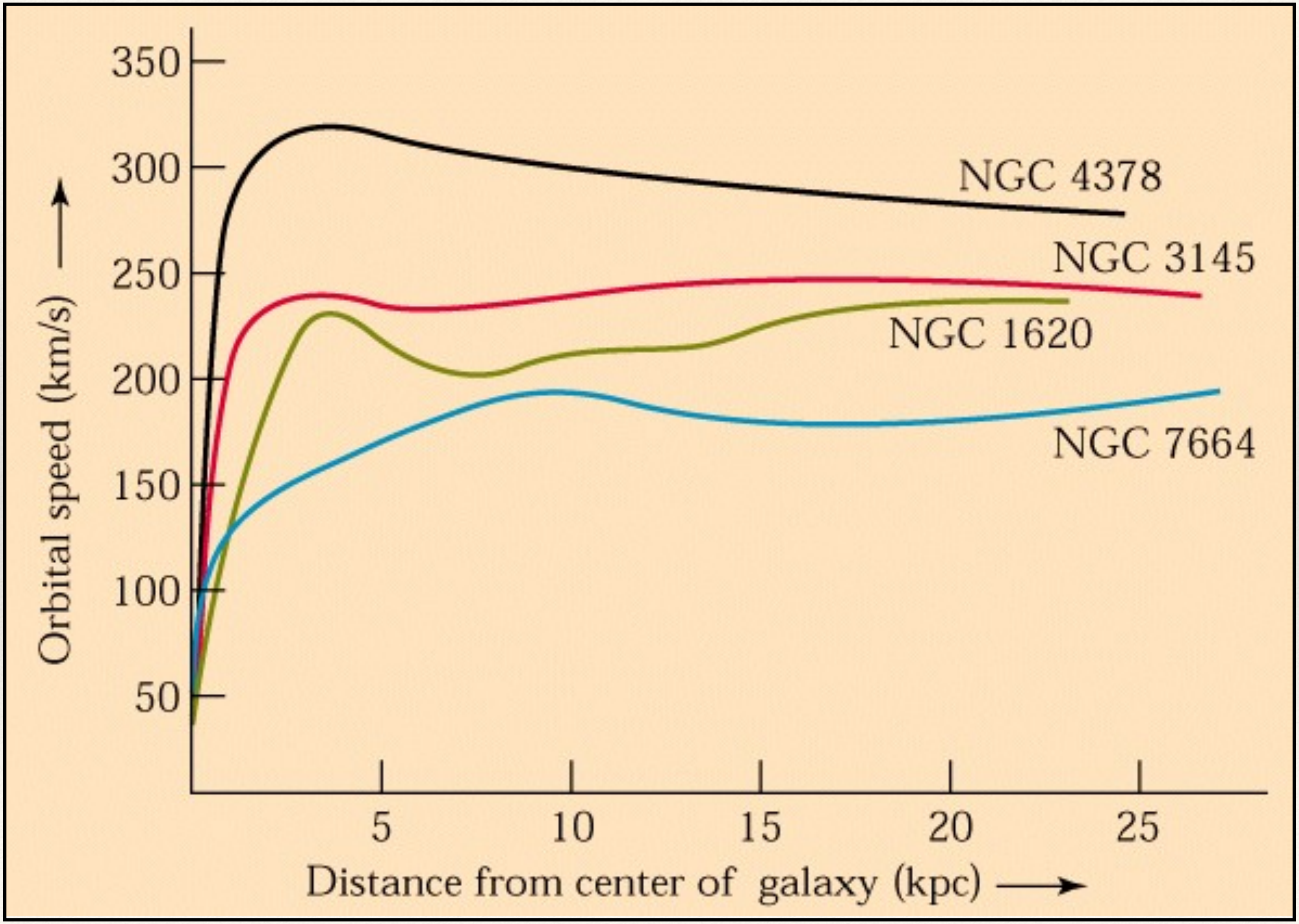
$$\text{Poisson Eqn} \quad \rho(r) = \frac{\nabla^2 \Phi}{4\pi G}$$

Integral Form

$$\text{Given} \quad \rho(r) \to \Phi(r)$$

$$\text{Integral Form} \quad \Phi(r) = -G \int d^3r' \frac{\rho(r')}{|r' - r|}$$

Rotation Curves of Spiral Galaxies



Power-Law density profile.

Spherical power-law density profile

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha}$$

Interior Mass profile.

$$M(r) = \frac{4\pi r_0^{\alpha}}{3-\alpha} r^{3-\alpha}$$

Circular velocity : For Flat rotation curve.

$$v_{\rm circ}^2(r) = \frac{GM(r)}{r} = \frac{4\pi G r_0^{\alpha}}{3-\alpha} r^{2-\alpha}$$

$$\text{For } \alpha = 2 \rightarrow v_{\rm circ} = \text{ const}$$

Other Potentials.

Hernquist Potential $$-\frac{GM}{r+b}$$

Plummer Potential $$-\frac{GM}{\sqrt{r^2+b^2}}$$

Jaffe Potential $$\frac{GM}{b}\ln\left(\frac{r}{r+b}\right)$$