Two-Shell Ionospheric Model for Indian Region: A Novel Approach

Ashish K. Shukla, Saurabh Das, Neha Nagori, M. R. Sivaraman, and K. Bandyopadhyay

***Abstract*—In the U.S. Wide Area Augmentation System, European Geostationary Navigation Overlay Service, and Indian Global Positioning System Aided Geo Augmented Navigation, a near real-time grid-based single-shell model is proposed to correct the ionospheric delay at the user aircrafts. The single-shell model is based on the assumption that the whole ionosphere is compressed at a fixed altitude at 350 km. This assumption may not be appropriate for the Indian region, which falls in the Equatorial Ionospheric Anomaly belt. In this paper, a two-shell model which incorporates two different shells, at 300- and 500-km altitudes, having different weights at different time domains has been implemented. A statistical comparison between single- and two-shell models has been done for all quiet days of year 2005. Based on the results, it is observed that there is at least 60% improvement in the performance of the two-shell model in comparison to the single-shell model for the Indian region.**

***Index Terms*—Grid-based model, ionosphere, total electron content (TEC), two shell.**

I. INTRODUCTION

SATELLITE-BASED Augmentation Systems (SBAS) for airline navigation are currently under development worldwide. The U.S. was the first country to develop and demonstrate the Wide Area Augmentation System (WAAS). Like the U.S. WAAS and other SBAS such as the European Geostationary Navigation Overlay Service [1], [2], the Indian Space Research Organization (ISRO) is also involved in the development and establishment of a full complement of WAAS over Indian airspace, called Global Positioning System (GPS) Aided Geo Augmented Navigation (GAGAN), in collaboration with the Airports Authority of India (AAI). The primary goal of SBAS is to enhance the accuracy and integrity of user position estimates based upon GPS measurements.

GPS satellites transmit signals on two L-band frequencies, namely, L1 at 1575.42 MHz and L2 at 1227.60 MHz. Time delay, due to the ionosphere when the signal passes through it, is an important aspect in the satellite ranging accuracy through GPS and is known as the ionospheric error. In the absence of the selective availability in GPS, the ionosphere is the largest source of error for single-frequency users of GPS.

There are many models available for the correction of this error such as Klobuchar [3], grid-based [4], [5], and tomography [6]–[8] models. One of the most popular models used in WAAS for this purpose is the single-shell model of the ionosphere. Currently, the ionospheric models used in SBAS rely on the single-thin-shell approximation [4], [5], which assumes that the whole ionosphere is compressed as a thin shell at a fixed altitude at 350 km. It, therefore, reduces a 3-D problem to a 2-D one [4], [5]. Because of the aforementioned assumption, the conversion of slant total electron content (STEC) to vertical total electron content (VTEC) using a mapping function at 350-km height can cause some error in the VTEC estimation. This error would be more if there is large variation of the vertical distribution of electron density.

WAAS derives slant ionospheric delay error and confidence bounds from estimates of vertical delay modeled on a grid at regularly spaced intervals of latitude and longitude ($5^\circ \times 5^\circ$). The vertical delay estimates at each Ionospheric Grid Point (IGP) are calculated from a planar fit of neighboring delay measurements. The product of the interpolated value at user position and the user's thin-shell obliquity factor provides an estimate of the user's ionospheric delay [9].

The single-shell model is a very simple model and is able to estimate the ionospheric corrections under nominal (quiet) ionospheric conditions up to sufficient accuracy at midlatitudes. However, the accuracy of the SBAS systems based upon the thin-shell model suffers due to the presence of complex ionospheric structure, high delay values, and large electron density gradients at low latitudes, particularly over the Indian region, due to presence of the Equatorial Ionospheric Anomaly (EIA) [10]. Thus, in order to capture the 3-D variation of the equatorial ionosphere, a two-shell approach was proposed [11]. It is found that the two-shell assumption can naturally model the vertical movement of the electron content by assigning the relative distribution of the electron content between the two shells and, thus, incorporates a better spatial variability than the single-shell model. The authors also felt the necessity of dealing the large-scale models with the fourth or temporal dimension. This is an important aspect keeping the fact in mind that the temporal variation of the ionosphere over India and in low-latitude region is appreciably large. Thus, to provide the two-shell model more sense, a novel approach has been adopted in this paper, which incorporates a two-shell model having two different shells, at 300- and 500-km altitudes, incorporating different weights for each shell at different time domains.

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A. K. Shukla and M. R. Sivaraman are with the Space Applications Center, Indian Space Research Organization, Ahmedabad 380015, India (e-mail: ashishs@sac.isro.gov.in).

S. Das is with the Space Applications Center, Indian Space Research Organization, Ahmedabad 380015, India, and also with the University of Calcutta, Calcutta 700073, India.

N. Nagori is with the University of Southern California, Los Angeles, CA 90089 USA.

K. Bandyopadhyay was with the Space Applications Centre, Indian Space Research Organization, Ahmedabad 380015, India. He is now with the Indian Institute of Technology, Kharagpur 721301, India.

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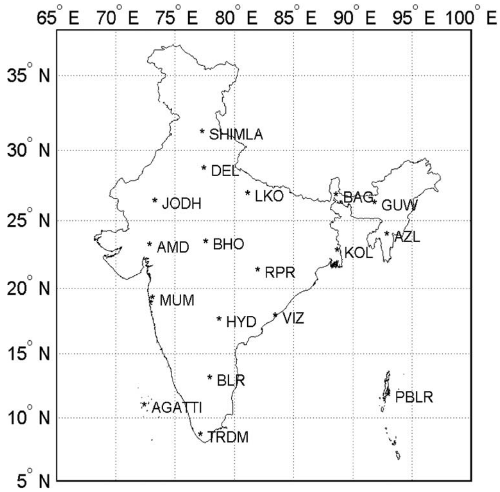

Fig. 1. Map of the locations of the stations collecting GPS data for GAGAN.

II. DATA COLLECTION AND PREPROCESSING

To carry out the comparison and validation of single- and two-shell models, total electron content (TEC) data have been collected from 18 stations having dual-frequency receivers, spread all over the Indian region. Locations of the stations are shown in Fig. 1. These stations are selected nearly at the centers of the 18 $5^\circ \times 5^\circ$ grids over India. The GPS data have been collected above the cutoff elevation angle of 15° from each of the 18 stations and used to calculate TEC values for this paper. It has been assumed that the 15° threshold can effectively minimize the multipath and other noises due to low elevation. TEC has been calculated from dual-frequency pseudorange data collected over the 1-min interval.

Before using the TEC data, receiver bias has been removed by using the Kalman Filter technique. As daily variation of the receiver bias is very small, it is estimated once in 24 h and applied for that day. Satellite bias correction has also been applied. Satellite bias includes differential P_1–P_2 code bias and the differential P_1–C_1 code bias, which can be estimated and measured separately. Differential P_1–P_2 code bias can be determined from the GPS broadcast differential group delay $({}^{\tau}GD)$ values by calculating GPS offset time and mean ${}^{\tau}GD$ values. A method of estimating P_1–P_2 bias has been studied, and a working algorithm has been developed. The P_1–C_1 bias values are available on the CODE website (http://www.aiub.unibe.ch/ionosphere.html). These values are used, and after converting into TEC units, correction has been applied. Applying all these interfrequency biases of satellite and receiver, the absolute true STEC from the receiver is obtained. Detailed description of preprocessing is beyond the scope of this paper and can be obtained from [12].

III. VARIATION OF DELAY DUE TO IONOSPHERE OVER INDIAN REGION

GPS signal delay caused by the ionosphere is directly proportional to the number of free electrons along the ray path signal. TEC along the ray path of the signal from the satellite to the receiver may be expressed as the path integral of the electron density along the line of sight [13]. Hence, to develop any ionospheric correction model over the Indian region, the knowledge of variation of the ionosphere over this region is of utmost importance. To have a clear view of temporal variation of TEC over the Indian region at different latitudes, the monthly mean plots of the diurnal variation of vertical delay of six stations, Trivandrum (8.48° N), Bangalore (12.95° N), Hyderabad (17.45° N), Bhopal (23.28° N), Delhi (28.58° N), and Shimla (31.08° N), have been shown in Fig. 2. These stations are situated such that they fall along a longitudinal belt around 77° and extending all the way from magnetic equator (Trivandrum) to the EIA crest region (Bhopal) and beyond (Delhi and Shimla).

Plots are shown for three months, namely, March, July, and December, which belong to three different seasons: equinox, summer, and winter, respectively. From Fig. 2, it can be observed that the maximum vertical delay for these months varies up to 8 m, and the peak is around 0900 h UT or 1430 h Indian Standard Time (IST). Thus, from the knowledge of variation of ionosphere over the Indian region (Fig. 2), it can be noted that ionospheric delay varies significantly with time, and the variation of delay is different for different times.

It can also be observed that for all the stations selected, delay is more in the months of March and July in comparison to the month of December. This is an expected behavior for equatorial regions [14]. This section gives a fair view of the actual temporal variation of delay (vertical) values for different latitudes over the Indian Equatorial region. This analysis will be important for error statistics discussed in Section VI. Another important aspect is the variation of maximum electron density with altitude over the Indian region, which has been discussed by Rama Rao *et al.* [15]. This forms the basis of our novel approach in the two-shell model, as described in the next section.

IV. NOVEL APPROACH IN THE TWO-SHELL MODEL

In a single-shell model, it has been assumed that the whole ionosphere is compressed as a single shell at 350-km altitude [5]. This height is approximately the height of maximum electron density and is assumed to be invariant in the single-shell model. Over the Indian region, the altitude of maximum electron density varies approximately from 300 to 500 km [15]. Hence, assuming the ionosphere as a single shell at 350 km would be inappropriate over the Indian region. Hence, a two-shell model with the ionosphere assumed to exist at two different heights, as two shells, may be a better choice, as suggested by Wu *et al.* [11].

In our two-shell model, we have assumed the ionosphere as having two shells at 300- and 500-km altitude, respectively, as shown in Fig. 3.

To account for the temporal variations of vertical distribution of electron content, we have assigned weights to TEC values at these two shells, using the study of variation of maximum vertical electron density distribution over the Indian region from ionosonde measurements [15]. In the Indian region, the height of maximum electron density is around 300 km from 0000–0800 h IST and goes up to 500 km from 0900–2300 h IST [15]. Hence, from 0000–0800 h IST, the contribution to TEC from lower shell (at 300 km) is taken as twice that of the upper shell (at 500 km), such that the sum of the weights remains unity, and vice versa from 0900–2300 h IST.

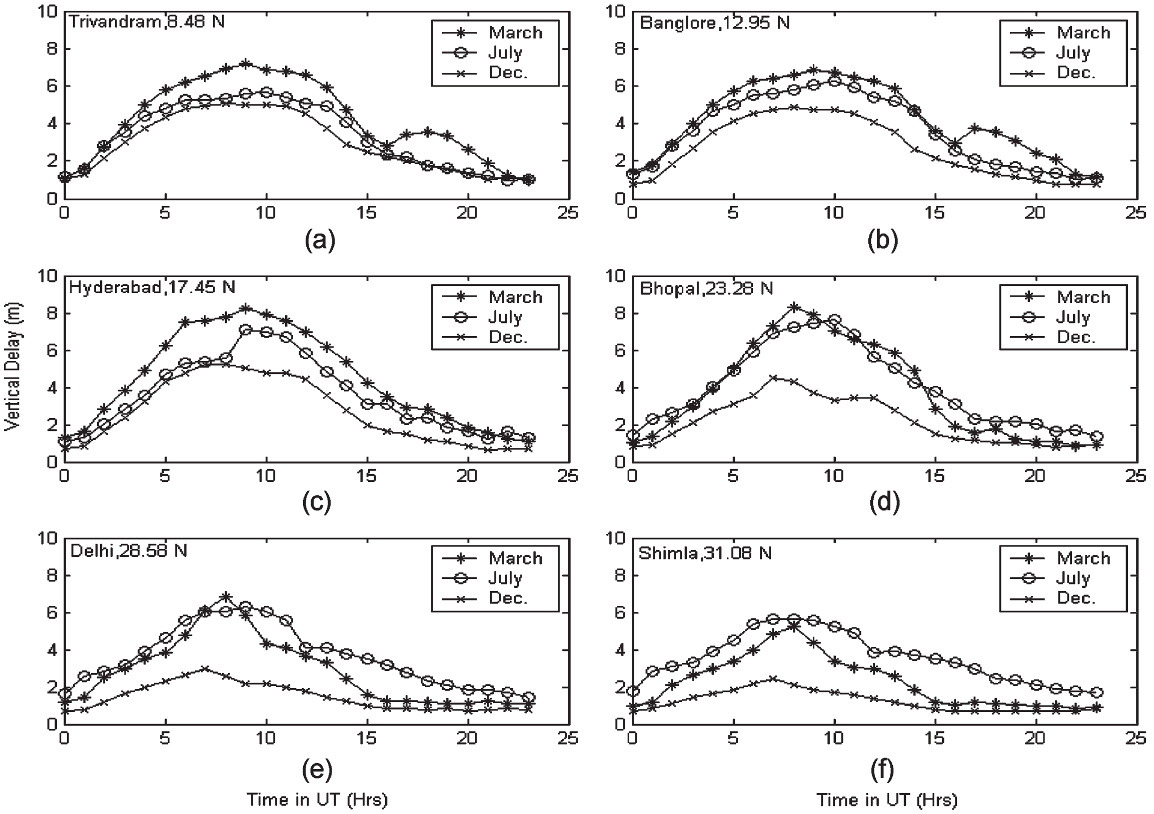

Fig. 2. Monthly mean plots of diurnal variation of vertical delay (in meters) of six TEC stations located at different latitudes over India.

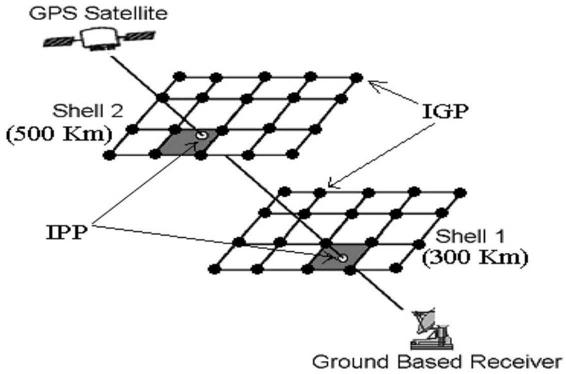

Fig. 3. Ray piercing through two thin shells at 300 and 500 km.

For this paper, the region of consideration (the Indian landmass) has been taken as 5° to 35° in latitude and 70° to 95° in longitude with a regular spacing of 5° in both the directions.

V. Method of Solution

For the estimation of vertical ionospheric delay values at IGP, for both single- and two-shell models, a planar fit algorithm has been used in this paper. In this proposed two-shell model, thin ionospheric shells are assumed at the mean heights of 300 and 500 km from the surface of Earth, between which vertical region maximum peak electron density is expected to vary. Before applying the planar fit algorithm for the two-shell model, weights have been assigned to STEC values for each shell for different times as described in Section IV of this paper. Afterward, STECs at 300 and 500 km have been converted into VTECs by dividing them by respective mapping functions at the two heights. Mathematically, conversion from slant to vertical TEC is expressed as

$$\text{TECvertical} = \frac{\text{TECslant}}{M(E,h)} \tag{1}$$

where

$$M(E,h) = \left[1 - (R_{\mathrm{e}} \cdot \cos E / R_{\mathrm{e}} + h)^2\right]^{-1/2} \tag{2}$$

is the mapping function [16]–[18]. Here, E, h, and R_{e}, respectively, denote elevation angle, maximum electron density altitude, and radius of the Earth.

The planar fit technique for the single-shell model [4] has been extended for two shells and is discussed in the following section.

A. Planar Fit Method

With the planar fit method, delay variations on a smoothened surface grided by IGPs are obtained by averaging highly variable spatial data from delay values at ionospheric pierce points (IPPs), as shown in Fig. 4. In the region around a particular IGP (for example, kth), the ionospheric delay about that kth IGP $(I_{\mathrm{v,IGP},k}(x,y))$ using planar fit for the first shell at 300 km is given as

$$I_{\mathrm{v,IGP},k}(x,y) = a_0 + a_1 x + a_2 y. \tag{3}$$

Here, a_0 represents the exact vertical delay at the kth IGP, a_1 and a_2 represent delay variation along X- and Y-axes,

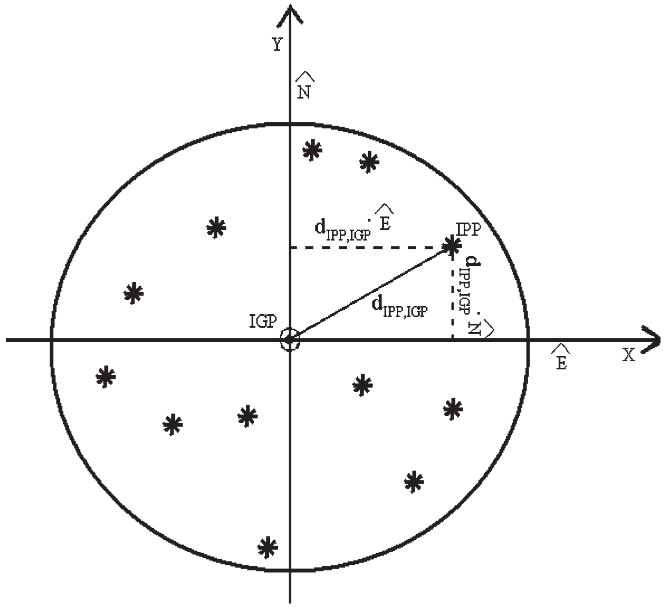

Fig. 4. Description of planar fit technique.

respectively, of a Cartesian coordinate system and has been shown in Fig. 4 above.

Here, x-axis of IGP aligns with the east direction and Y-axis aligns with the north direction. "*" denotes IPPs around the kth IGP, and $d_{\mathrm{IPP}_N,\mathrm{IGP}}$ is defined as a 2-D vector from the IGP to Nth IPP. $\hat{E}$ and $\hat{N}$ are the unit vectors in the eastward and northward directions, correspondingly.

The solution for planar coefficients (a_0, a_1, a_2) is obtained by the weighted least square method and is given as

$$[a_0 \quad a_1 \quad a_2]^{\mathrm{T}} = \left[(G.W.G^{\mathrm{T}})^{-1}.G.W.I_{\mathrm{v,IPP}}\right] \tag{4}$$

where $I_{\mathrm{v,IPP}}$ is the $N \times 1$ matrix of vertical delays at IPPs surrounding an IGP within a neighborhood of 8° radius [16], obtained from slant delay measurements using GPS receivers.

W is the weight matrix, and its inverse is given in (5), shown at the bottom of the page.

Here, optimal weights for IPPs are calculated based on the residual errors. The residual error is obtained by using the correlation structure, which includes measurement variances of IPP $(\sigma^2_{I_{\mathrm{v}},\mathrm{IPP}_i})$, decorrelation factor $(\sigma^2_{\mathrm{decorr}})$, and biases $(\sigma_{\mathrm{bias},i,j})$ forming weight matrix (W) [4].

G is the $3 \times N$ observation matrix for the kth IGP, and its transpose G^{T} is

$$G^{\mathrm{T}} = \begin{bmatrix} 1 & d_{\mathrm{IPP}_1,\mathrm{IGP}} \cdot \hat{E} & d_{\mathrm{IPP}_1,\mathrm{IGP}} \cdot \hat{N} \\ 1 & d_{\mathrm{IPP}_2,\mathrm{IGP}} \cdot \hat{E} & d_{\mathrm{IPP}_2,\mathrm{IGP}} \cdot \hat{N} \\ 1 & d_{\mathrm{IPP}_3,\mathrm{IGP}} \cdot \hat{E} & d_{\mathrm{IPP}_3,\mathrm{IGP}} \cdot \hat{N} \\ \vdots & \vdots & \vdots \\ 1 & d_{\mathrm{IPP}_N,\mathrm{IGP}} \cdot \hat{E} & d_{\mathrm{IPP}_N,\mathrm{IGP}} \cdot \hat{N} \end{bmatrix}. \tag{6}$$

Since the IGP is located at the origin in the local coordinate system, vertical delay at the IGP can be obtained by determining planar coefficient a_0, making other coefficients zero.

Finally, the vertical delay estimate at the kth IGP at 300-km altitude is given by [4]

$$I_{\mathrm{v,IGP},k} = [1 \quad 0 \quad 0].\left[(G.W.G^{\mathrm{T}})^{-1}.G.W.I_{\mathrm{v,IPP}}\right]. \tag{7}$$

A similar approach for the second shell at the altitude of 500 km has been adopted to get the vertical delay estimate at the respective kth IGP.

Total vertical delay is the linear sum of the delays obtained by using the planar fit technique at both the shells at respective IGPs.

The following section provides the error estimates in reconstructed STEC from both single- and two-shell models.

B. Validation Methodology

In this paper, data used for the models are STEC measurements from all available satellites from the 18 locations, as already described in Section II of this paper. Using this data, ionospheric delay values at IGPs have been determined from both single- and two-shell models. Afterward, assuming each IPP as a user, the STEC at IPPs has been reconstructed.

Yearly average root mean square error (rmse) in STEC has been estimated for all quiet days of the year 2005, which includes 72 test days. Choice of quiet days has been done on the basis of the geomagnetic Ap index. For the present analysis, an Ap index less than 50 is considered, which covers days with geomagnetic activity up to minor storms and excludes major and severe storms [19]. The following formula has been used for the calculation of the rms errors in STEC:

$$\mathrm{rmse}_{\mathrm{STEC}} = \sqrt{\frac{\sum_{i=1}^{N} \left(\mathrm{STEC}_{\mathrm{m}}(i) - \mathrm{STEC}_{\mathrm{r}}(i)\right)^2}{N}} \tag{8}$$

where $\mathrm{STEC}_{\mathrm{m}}$ is the measured (true data) STEC, $\mathrm{STEC}_{\mathrm{r}}$ is the reconstructed STEC, and N is the total number of IPPs. Finally, Slant TEC (in TEC units) values are converted to slant delays (in meters).

VI. RESULTS AND DISCUSSION

The results of comparison of single- and two-shell models are shown in Fig. 5. We have determined rmse at different times of a day for all quiet days of 2005, which includes 72 test days from 18 locations spread over the Indian region. The average

$$W^{-1} = \begin{bmatrix} \sigma^2_{I_{\mathrm{v}},\mathrm{IPP}_1} + \sigma^2_{\mathrm{decorr}} & \sigma_{\mathrm{bias},1,2} & \cdots & \sigma_{\mathrm{bias},1,N} \\ \sigma_{\mathrm{bias},1,2} & \sigma^2_{I_{\mathrm{v}},\mathrm{IPP}_2} + \sigma^2_{\mathrm{decorr}} & \cdots & \sigma_{\mathrm{bias},2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{\mathrm{bias},1,N} & \sigma_{\mathrm{bias},2,N} & \cdots & \sigma^2_{I_{\mathrm{v}},\mathrm{IPP}_N} + \sigma^2_{\mathrm{decorr}} \end{bmatrix} \tag{5}$$

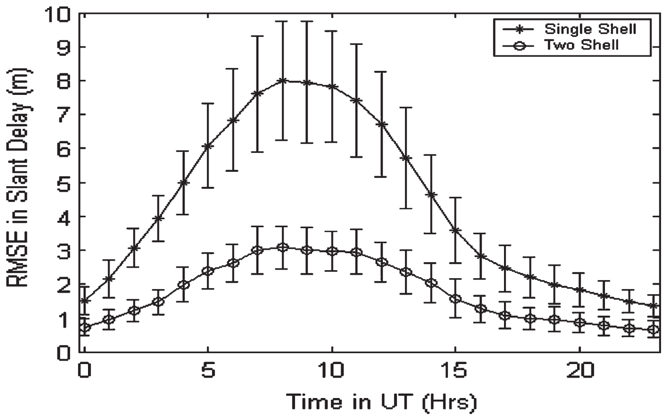

Fig. 5. Comparison of rmse in slant delay (in meters) from single- and two-shell models for all quiet days of 2005. Vertical bars represent standard deviation.

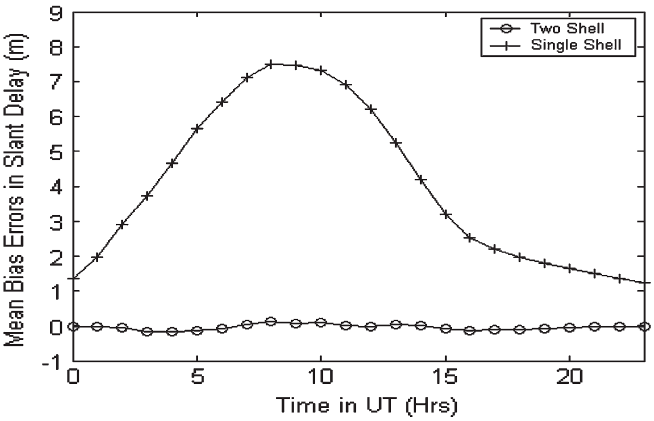

Fig. 6. Comparison of mean bias errors for single- and two-shell models.

error estimates along with the standard deviation values, shown by error bars, for all test days are plotted in Fig. 5. It can be seen clearly from Fig. 5 that the rmse is least for the two-shell model in comparison to the single-shell model for all the times.

In the case of the two-shell model, maximum rmse is only up to 3 m, while in the case of the single shell the error goes up to 8 m, which is of quite high magnitude. This clearly indicates that there is at least 60% improvement in the performance using the two-shell model for the Indian region. Furthermore, it has been observed that during the peak hours of the day, when ionospheric activity is increased, the two-shell model seems to capture the variation very well, giving small rms errors in comparison to the single-shell model. It can also be observed from the standard deviations represented by vertical bars, the uncertainty in estimating errors is comparatively less using the two-shell model. Improvement in the performance of the two-shell over the single-shell model may be due to the fact that the two-shell model well incorporates the variation of maximum electron density with time from 300- to 500-km altitude.

Furthermore, incorporating bias errors into model verification studies provides a different, additional way to look at the model output. Mean bias error (MBE) has been calculated using the following formula:

$$\mathrm{MBE} = \frac{\sum_{i=1}^{N} \mathrm{STEC}_{\mathrm{m}}(i) - \mathrm{STEC}_{\mathrm{r}}(i)}{N}. \qquad (9)$$

Thus, in Fig. 6, a comparison of bias errors for both the single- and two-shell models has been shown.

Comparison shows that the two-shell model provides a better estimate of TEC than the single-shell model over the Indian region for all times. From the figure, the overall trend shows that the two-shell model is neither overestimating nor underestimating the delay values, and most of the time, bias errors remain near zero.

Choice of testing under quiet conditions was made only to exclude the effects due to various abrupt variations derived from enhanced geomagnetic activities. This model may further be tested with more years data and for disturbed days as well.

VII. CONCLUSION

To incorporate the effect of variation of maximum electron density with altitude over the Indian region, a novel two-shell approach has been proposed. This approach also includes the temporal variation assigning different weights to STEC values for each shell for different time domains. The model has been tested for all quiet days of year 2005 (72 days), and a statistical comparison between the proposed two-shell model and existing single-shell model has been done. Based on the results, it is observed that there is at least 60% improvement in the performance of the two-shell model in comparison to the single-shell model. This paper shows that the two-shell model may be a better choice for the Indian region.

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Ashish K. Shukla received the Masters and Ph.D. degrees in applied mathematics from Lucknow University, Lucknow, India, in 1997 and 2003, respectively.

He is currently a scientist in the SATCOM and IT Applications Area, with the Indian Space Research Organization, Ahmedabad, India. His research interests include propagation modeling in the ionosphere and troposphere and GPS applications.

Saurabh Das received the M.Sc. degree in physics from the Indian Institute of Technology, Roorkee, India. He is currently working toward the Doctoral degree in satellite communications at the University of Calcutta, Calcutta, India.

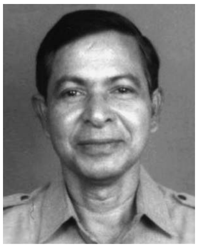
He is currently a Senior Research Fellow with the Indian Space Research Organization, Ahmedabad, India. His area of interest includes signal propagation through the ionosphere and troposphere.

Neha Nagori received the B.E. degree in electronics and communications from Sardar Patel University, Gujarat, India, in 2004. She is currently working toward the M.S. degree in digital signal processing at the University of Southern California (USC), Los Angeles.

From 2005 to 2007, she was with the Systems and Applications Group, Space Applications Centre, Ahmedabad, India.

M. R. Sivaraman received the Ph.D. degree in physics from University of Gujarat, Ahmedabad, India.

He was Deputy Project Director of GPS Aided Geo Augmented Navigation-Technology Demonstration System. He is currently a Senior Ionospheric Scientist with the Indian Space Research Organization, Ahmedabad, India. His research interest includes study of ionospheric phenomenon over low latitudes.



K. Bandyopadhyay received the Ph.D. degree in electronics and telecommunications engineering from Jadhavpur University, Calcutta, India

He was a Senior Scientist/Engineer with the Indian Space Research Organization (ISRO), Ahmedabad, India, an Associate Project Director of GPS Aided Geo Augmented Navigation-Technology demonstration System and also heading various other programs with ISRO, and currently a Professor with the Indian Institute of Technology, Kharagpur, India.