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# Site Diversity Prediction at a Tropical Location From Single-Site Rain Measurements Using a Bayesian Technique

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**Abstract** Site diversity is one of the few fade mitigation techniques that are actively explored to overcome the serious fading condition encountered in high-frequency satellite communication, particularly over tropical regions. However, the study of site diversity performance is limited over tropical regions due to unavailability of simultaneous fade measurements from multiple locations as well as the lack of data on spatial rain pattern from radar measurements. We propose here a copula-based Bayesian approach to study the site diversity performance utilizing only a single location's drop size distribution measurement. Study of diversity performance has been done for Ahmedabad, India, utilizing 3 years of disdrometer data. Effects of frequency, elevation angle, and site separation on diversity gain are investigated using the proposed method. Results are compared with two ITU-R models, site diversity results from Italy, a temperate region, and Singapore, another tropical location. The spatial correlation obtained through this technique is also compared with radar observations in Oklahoma, a totally different climatic regime. The simple ITU-R model is found to be underpredicting the diversity gain significantly for the present location whereas the explicit ITU-R model shows overprediction above 8 dB for single-site rain attenuation values. Comparison with measurements at Italy shows similar diversity gains for low single-site attenuation values. The results reported for Singapore matches well with the present method. The study indicates room for improvement in the present ITU-R model considering the different nature of tropical rain and that the proposed method is suitable for such a purpose.

**Key Points:**
- Site diversity prediction is based on single disdrometer measurements
- Copula-based Bayesian approach is used for spatial field generation
- Comparison with ITU-R model shows significant differences

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## 1. Introduction

Rain attenuation is a serious consideration in operational satellite communication systems utilizing frequencies above 10 GHz, particularly over tropical regions due to high rain rates and frequent occurrences of rain events (Das et al., 2010; Green, 2004; Ippolito Jr, 2008; Panagopoulos et al., 2004). Conventional fade mitigation techniques such as adaptive power control may not be an effective tool to overcome the rain attenuation in Ka/V band in which fading can be in excess of 20 dB for substantial percentage of time in a year (Green, 2004; Panagopoulos et al., 2004; Roy et al., 2012). In this scenario, diversity schemes are explored as alternatives to avoid such high fade situations by exploiting the space-time variability of rainfall.

Rain is not a homogeneous process either in space or time. Utilizing these variabilities, several diversity fade mitigation techniques are proposed, such as time diversity, site diversity, satellite diversity, and frequency diversity (Castanet et al., 1998; Ippolito Jr, 2008; Jeannin et al., 2007; Panagopoulos et al., 2004). These techniques rely on the fact that the joint probabilities of encountering the same fade at two distinct points either in space or in time are much less than the probability of encountering the same fade at a single point (Capsoni & Matricciani, 1985; Das et al., 2013; Maitra et al., 2009). The site diversity is one of the most preferred techniques among these diversity methods that utilizes the spatial variability of rain to its advantages (Castanet et al., 1998; Hodge, 1982; Jeannin et al., 2012; Yeo et al., 2011). Since the high rain rate is usually associated with convective rain, which is characterized by events of short duration and of limited extent, site diversity could be very useful for tropical regions (Das et al., 2010, 2013; De et al., 2016; Maitra et al., 2009; Yeo et al., 2011).

There are limited studies of site diversity performances over tropical regions due to unavailability of simultaneous measurements from two or more satellite signal-receiving stations. The ITU-R (2015) site diversity prediction models are commonly used for estimating the diversity gain in absence of experimental measurement. Even where the local measurements are available, only the rain rate probability values are used for estimation of the site diversity gain without taking consideration of the local spatial or microphysical behavior of the rain. The explicit ITU-R model requires joint rainfall probability estimation using data from two or more locations, which are often difficult to obtain in a practical scenario. The simplistic model is an adaptation of Hodge (1982) model and requires measurements only from single station. ITU-R models are developed mostly based on measurements from temperate regions, and the performance of the models can thus be questionable over tropical regions (Shukla et al., 2010; Yeo et al., 2011). Moreover, the issue of local variability of meteorological factors remains even in temperate regions.

The inadequacy in ITU-R models are often due to the following reasons:

1. The spatial and microphysical structure of rain varies for different locations. On large scales, the ITU-R modeling acknowledges that such differences exist; however, it does not attempt to address local meteorological variations.
2. The structure of rain are largely different in different types of rain, particularly the size of rain cells. ITU-R does not consider this variation explicitly but rather models the rain cell sizes in terms of rain rates, which again lacks the information of local rain characteristics.

Considering the limited availability of experimental measurements, it is, therefore, imperative to explore new methods that can be used to estimate the site diversity performance incorporating the local rain characteristics. Several attempts have been made in the past to predict the site diversity based on statistical properties of the rain rate (Livieratos et al., 2014; Panagopoulos et al., 2005) as well by considering the physical nature of rain cell and vertical rain distribution (Bosisio & Riva, 1998; Feral et al., 2003; Luglio et al., 2002; Matricciani, 1994, 1996). Alternatively, many studies are based on the spatial rain structure measurements from weather radar and rain gauge networks (Capsoni & Matricciani, 1985; Matricciani, 1994; Nagaraja & Otung, 2012; Pan & Bryant, 1992; Shukla et al., 2010; Yeo et al., 2011). Although radar studies are useful when direct measurements using satellites are not available, radar coverage over the globe is also very limited. Radars usually estimate mean values of quantities over large domains. However, remote sensing techniques cannot measure the physical quantity directly but can only retrieve the parameters using algorithms under some assumptions. Retrieved rain parameters can thus be affected by beam spatial filtering (Jameson & Larsen, 2016a; Jameson, 2016, 2017). Further, one does not always have access to a radar and establishing a large rain gauge network is not an easy task.

One important research thrust over the last decade is to address this issue by upscaling the point measurements to larger areas. In Jameson (2015) a method is presented for converting time series of 1-min disdrometer observations to the spatial patterns while maintaining the statistical and meteorological properties of the observed rainfall. Here we further extend this approach to estimate and predict the site diversity performance based on drop size measurements from a single location. The time series of raindrops measured by a disdrometer is first transformed into the spatial rainfall distribution under the framework of copula statistics (Jameson, 2015), which in turn provides the spatial distribution of the attenuation fields. The site diversity can then be studied for different site separation lengths and orientations.

The whole purpose of the present approach is to account for the obvious dependence of different meteorological and climatological conditions at different locations. The contribution of the present work is to offer a method for tuning the general ITU-R approach to regional differences using a single-site drop size measurements. The technique generates the spatial fields that reflect the local microphysical and spatial behavior of the rain. Further, the technique takes care of the different spatial structure associated with different rain types. Hence, this approach provides a more realistic estimation of site diversity at any location by considering the local rain characteristics using single-site measurements.

The novelty of such an approach is that for each point, the cumulative distribution remains invariant since the statistical properties of original time series is preserved. However, the method simultaneously generates spatial features that closely resemble those actually observed at that location. Since the proposed method depends only on the drop size distribution (DSD) characteristics of a single location, the method can be extended to study the diversity performance at any location. The information at each drop size is crucial in the proposed technique because the correlation among drops are related to the microphysical structure of rain (Jameson & Larsen, 2016b). Hence, this method implicitly accounts for the rain types and local rain characteristics. With adjustments, it should be possible, in principle, to convert such a rainfall rate time series into a spatial pattern using the proposed method. However, using the time series of DSD for such purpose is still advantageous because it also provides the spatial variation of DSD for detailed calculations of any desired parameter such as attenuation without having to assume any a priori relations.

We have applied this methodology using 3 years of disdrometer measurements at Ahmedabad, a tropical location of India. The spatial rain correlations estimated using the proposed method at Ahmedabad are compared with measurements in Oklahoma, using 2-D video disdrometer network and radar (Bringi et al., 2015). Since diversity performance from only a few tropical locations are available in literature, we have to limit our comparisons with the reported results from Singapore using weather radar data(Yeo et al., 2011). Site diversity estimated using network of rain gauges at two locations of Italy, another temperate region, are also compared for small distances (Matricciani, 2003). In addition, for completeness, the proposed method is also compared with the ITU-R diversity gain models (ITU-R, 2015), which often appears to perform satisfactorily over temperate regions (Callaghan et al., 2008).

In the next section, a description of the experimental data collected at Ahmedabad is presented along with a description of the procedure used to convert the time series into spatial fields. This is followed by the analysis methodology. The results are presented in section 3. In section 4, the assumptions used in the technique are discussed, while the conclusions are presented in section 5.

## 2. Data and Methodology

Three years of DSD measurements during 2005 – 2007 from Ahmedabad, India (23.04°N, 72.38°E) is utilized in the present study to simulate the site diversity performance. The rain events are used to produce spatial rain fields that can then be transformed into attenuation fields according to the procedure outlined in ITU-R (2015) model. In the following subsections, the conversion processes and analysis methods are discussed in detail.

### 2.1. Data Description

Distinct rain events are first extracted from the database. Rain events are defined when rain occurs continuously at least for 10 min and having maximum rain rate above 5 mm/hr. An impact-type disdrometer manufactured by Disdromet (RD-80) is used to characterize the DSD.

Raindrops that strike the sensitive Styrofoam surface of the disdrometer generate an electric pulse, related to the fall speed of the drops. Using the standard assumptions that the drops are falling with terminal velocity and using the empirical relationship of Gunn and Kinzer (1949), the number of drops of different sizes striking the surface can then be converted to the DSD, which in turn give the rain rate and other integral rain parameters.

The raindrops measured by the instrument in the range of 0.3 – 5 mm is further categorized into 20 classes. The instrument has a catchment area of 50 $cm^2$ and operates with 30-s temporal resolution. This type of disdrometer tends to underestimate the number of small drops when large drops are also striking the instrument, but then they are less important because of the mass difference between the large and small drops. Another issue with this type of disdrometer is acoustic noise, which is minimized by ensuring the proper installation. To reduce the uncertainty in DSD, 1-min integration times are also used in further processing. Interested readers can refer to the papers (Das et al., 2010, 2013; Maitra et al., 2009) to have further insight into the experimental details and characteristics of rain structure of Ahmedabad.

### 2.2. Conversion of Rain Time Series to Spatial Distribution

The distribution of the raindrop sizes is the key factor governing the rain-signal interactions as well the rain physics (Das & Ghosh, 2016). Any time series of DSD can be assumed to be just one sample from the set of multiple statistical processes, which govern the rain process. The mean values of the drop counts for different DSD time series of a location are thus random but correlated. These correlations in counts are, in fact, related to the scales of the precipitating system. The DSD time series can be converted into spatial patterns of detailed DSD at each grid point using a Bayesian approach for estimating the drop counts in combination with a copula technique and the root matrix method for generating 2-D correlations corresponding to the observations. This information can then be transformed into rain integral parameters and rain attenuation.

A Bayesian inversion method has been developed for estimating the probability distribution of the mean values of a series of observations. While details may be found in the references, briefly, one considers a range of mean values. Each observation can then associated with each of these mean values to some degree of probability based upon an assumed probability distribution for the observations (the so-called likelihood function). Over the entire set of observations, the probabilities at each mean value are then summed and normalized to unity to yield a final estimate of the probability distribution of the mean values themselves.

This inversion is insensitive to correlations. Specifically, the distribution of mean values, $C$, for particles of a specified size is given by Jameson (2007)

$$P(C|n, D) \propto \frac{C^n}{n!} e^{-C} \tag{1}$$

where $n$ represents the number of counts, $C$ is the distribution mean, and the vertical line indicates conditioning.

Here we assumed Poisson statistics for drop counts. It has been shown that this assumption is not critical for the present approach (Jameson, 2015; Sivia & Skilling, 2006). In fact, the distribution given by (1) changes with every new count. Overlaying each of these $P(C|n)$ for each observation of $n$ can therefore gives the full probability density function (pdf) of $C$ as shown in (2).

$$P(C|D) = \frac{\sum_{i=1}^{T} \frac{C_i^n}{n_i!} e^{-C} P(n_i|D)}{\sum_{C=0}^{C_{max}} \sum_{i=1}^{t} \frac{C_i^n}{n_i!} e^{-C} P(n_i|D)} \tag{2}$$

where $T$ denotes the total number of observations, $P(n_i|D)$ indicates the probability of observing $n$ raindrops of size $D$ at the $i$th instance. The denominator used to normalize the distribution.

Further, the raindrops are correlated among themselves. These correlations are derived naturally from the observations. Furthermore, these auto-correlation functions also give rise to the appropriate cross-correlation among different size classes when applied to a same random number field. Since this correlation is directly related to the meteorological structure (Jameson & Larsen, 2016b), it can also be regarded as a reflection of the associated physical processes. A priori information of the actual physical process is not needed to estimate the correlation function.

The rain process may involve many intermittent periods without any raindrops of particular type, especially in case of large drops. It is necessary to account for the fractional time, $F$, when there are no counts. The $P(C)$ is then adjusted accordingly using the $F$. Using the copula transformation technique, the distributions $P(C)$ of the mean values, $C$, are then computed, maintaining both the fractional empty time and the observed correlation function (Frees & Valdez, 1998; Genest & Mackay, 1986; Jameson & Kostinski, 2008; Nelsen, 2006).

To get the spatial distribution, first, an $m \times m$ correlated 2-D field is generated with zero mean, unit variance uniformly distributed random real numbers over [0,1] having the characteristic correlation appropriate to each drop size. These correlated variables are then inverted using the accumulated pdf to derive the correlated field of the desired variable. This is done for each drop size separately using the root matrix method (Jameson & Kostinski, 1999; Johnson, 1994). The copula technique provides the necessary transformation for calculating the correlated, accumulated pdf of the random variable.

The above mentioned procedure leads to realistic DSD at each grid points having the observed correlations and pdf properties including the empty fractions. These can then be transformed to rain integral parameters such as rainfall rate, specific attenuation, and radar reflectivity factor. A Matlab program using the methodology for processing a complete set of event data has been developed for such processing and available at http://saurabh-works.droppages.com/SDBC.html.

### 2.3. Conversion of Rain Rates to Attenuation Values

Rain field data generated by the method described in previous section is used with the procedure mentioned in ITU-R P. 618-10 (ITU-R, 2015) to estimate slant path rain attenuation. The rain attenuation is estimated at each grid point. The specific attenuation associated with rain rate at each grid point is estimated as per ITU-R P. 838-3 (ITU-R, 2005) and integrated along a hypothetical satellite link of a predefined elevation angle to get the total attenuation value suffered by the link. The portion of slant path length, which is under the rain region, is determined from rain height information. For the present location, it is assumed to be 4.5 km consistent with the tropical environment as per ITU-R P. 839-4 (ITU-R, 2013). The slant path length under the rain region is thus a function of elevation angle, and the total attenuation depends on the rain rate distribution along that slant path length.

The specific attenuation ($\gamma$) is estimated as follows: $\gamma = aR^b$ where $a$ and $b$ are the coefficients taken from ITU-R P. 838-3 (ITU-R, 2005) and is a function of frequency. Hence, the total attenuation depends on the frequency, elevation angle, and rain rate distribution. To study the performance of diversity scheme at Ahmedabad, simulations are performed with different values of these parameters as discussed next.

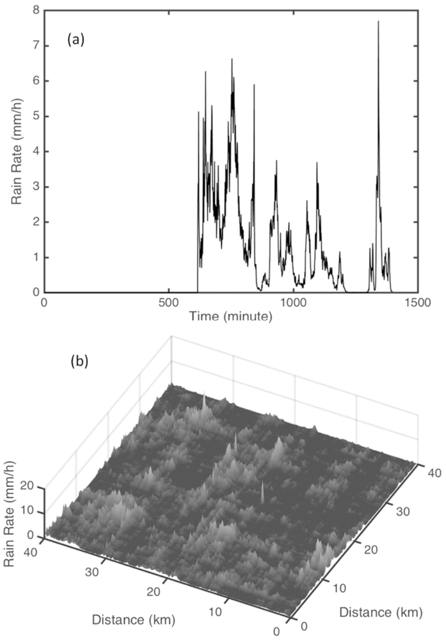

**Figure 1.** (a) Rain rate time series and (b) simulated spatial rain pattern of the same stratiform event.

### 2.4. Analysis Procedure

The distance between two grid points is directly related to the temporal resolution of the data, and the wind speed and direction, since it is assumed that the spatial field is generated by the motion of the storm. An average wind speed of 8 m/s is assumed in present work. Since 1-min temporal resolution DSD data are used for generation of the rain field, the grid separation turns out to be 0.48 km. We also assumed a unidirectional storm motion for simplicity.

For the present study, a fixed set of random number base of 50 × 50 points is chosen to develop the spatial field. Each grid point is identical in the sense that the spatial features are based on the statistical inversion from a single location's data. Therefore, instead of selecting any specific grid point as reference point, the diversity statistics are generated for each pair of points with varying distance. The average and standard deviation value of the parameters from all the pair of points are then utilized to compare the results with other methods. Thus, a robust estimation of site diversity performance is possible. For example, as the spatial field developed under the Bayesian method has the dimension of 50 × 50 grid points and each grid spacing represent a ~0.5-km distance, we can have a total of $\sim 49^2$ different combinations of pairs of points with 0.5-km separation for every time instant. Here we consider the pair of points only along one direction for simplicity.

The correlation of rain rate time series between every pair of points was first studied to identify the maximum distance up to which the correlation exists. Next, we simulate the slant path rain attenuation for each grid point assuming a hypothetical link to the GSAT-14 satellite at an elevation of 60° at 30 GHz. The Indian Space Research Organization launched geostationary satellite GSAT-14 at 74° latitude, which is carrying 20- and 30-GHz beacons to study the rain attenuation features over the Indian region. We also simulate the attenuation time series for hypothetical link at 20°, 40°, and 80° and for frequencies 10, 15, 20, and 25 GHz. As site diversity performance does not depend upon the signal polarization (Yeo et al., 2011), only horizontal polarization is used for the present study.

To analyze the performance of the proposed method, the diversity gain has been studied for varying distances, elevation angles, baseline orientations, and frequencies. The results are compared with ITU-R models. There are two models currently recommended by ITU-R. The explicit model of site diversity prediction is based on the physical model of Luglio et al. (2002), which estimates the joint attenuation exceedance probability from rain measurements at two sites. The other model (Hodge, 1982) is a simplistic one and uses only single-site rain measurements. A general comparison is performed with both models and the reported values of diversity gain from Singapore, another tropical location. However, since the present work utilizes only a single location's data, we limit further comparisons only with the simplistic model of ITU-R.

## 3. Results

### 3.1. Simulation of Spatial Characteristics of Different Rain Types

Figure 1a shows a typical rain event observed on 30 June 2005 at Ahmedabad. The event is characterized by low rain rate with long durations, which is usually associated with stratiform rain. This event has maximum rain rate about 10 mm/hr and spanned over more than 12 hr. The spatial pattern of this rain event generated by the Bayesian method shown in Figure 1b indicates that the rain is spread over a large area, which is as expected. In comparison, Figure 2a shows another rain event with high rain rates observed on 28 June 2005. This event is characterized by short duration and heavy rainfall. The maximum rain rate goes above 50 mm/hr, and this type of rain usually is part of a convective system. The spatial pattern of such rain is characterized by small rain cell sizes as efficiently simulated by the Bayesian method as shown in Figure 2b. The Indian region is frequented more often by convective rain than temperate regions. The rain attenuation due to such high rain rates is quite high.

The low rain stratiform event has much larger rain cell size than the high rain rate convective cells. Since tropical regions are usually characterized by the more convective type of rain, site diversity gain is expected to be higher in tropical regions than in temperate regions.

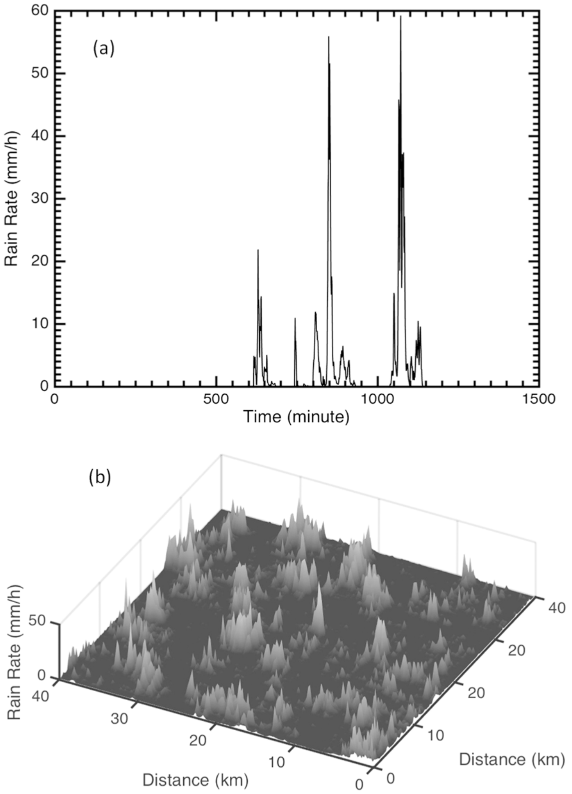

**Figure 2.** (a) Rain rate time series and (b) simulated spatial rain pattern of the same convective event.

### 3.2. Spatial Correlation of Rain

The site diversity technique utilizes the spatial inhomogeneity of rain to overcome the signal fading condition at two or more links separated by a distance. The Pearson correlation coefficients between the time series of rain measured at two distinct points can thus give an idea of the spatial variation of the rain. The spatial correlation ($\rho$) can be modeled (Jaffrain & Berne, 2012; Krajewski et al., 2003) with distance ($d$) in a three-parameter form as

$$\rho(d) = \rho_0 exp(-(\frac{d}{d_0})^F) \quad (3)$$

where $d_0$ denotes the decorrelation distance where the correlation falls $e^{-1}$ times of the initial value. The $\rho_0$ indicates initial correlation value, which is 1 when site separation is zero, that is, correlation with itself. $F$ is the shape parameter.

In Figure 3a, the variations of correlation coefficients with distances are shown for the previously shown two rain events. A continuous moving window of 100 samples from these rain events are used here to generate the spatial rain field. To avoid any additional correlation introduced due to the sampling from a single rain event, variable "random number base" is used in each iteration with a systematic gap of 5 min. This gives time series of rain rates at each grid point of that particular rain event. The grid locations that do not have any rain information are not considered for this analysis. Since all grid points are statistically identical, we estimated the correlation for all pairs of points with varying distances. The average correlation values for each separation distance are then plotted for the two events. From the figure, it can be seen that the proposed method can successfully capture the difference between the convective and stratiform types of rain solely based on the time series measured at a single location. The correlation falls slowly with distance in case of stratiform type of rain whereas it decreases rapidly with distance in case of convective type of rain. The modeled decorrelation distances are found to be 2.6 km for convective rain and 4.7 km for stratiform event. The value of the $F$ parameters are found to be 0.84 for convective rain and 0.52 for stratiform rain in case of Ahmedabad. The spatial correlation of different types of rain measured using NASA S-band polarimetric radar and 2-D video disdrometer at Oklahoma is also shown for comparison (Bringi et al., 2015). For the specific cases shown in

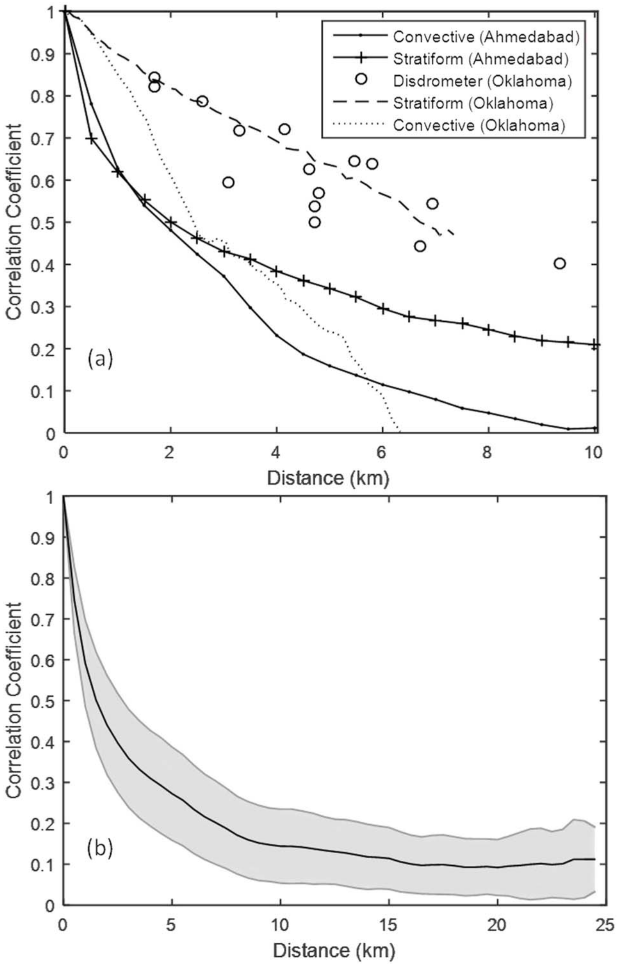

**Figure 3.** Spatial variations of correlation coefficients of rain rate for (a) different types of rain and (b) average of all rain events. Oklahoma points were taken from Bringi et al. (2015).

the figure, Bringi et al. (2015) reported the decorrelation and *F* values of 3.4 km and 1.52, respectively, for convective event and 7.5 km and 0.6, respectively, for stratiform event. In an earlier study, Thurai et al. (2012) obtained decorrelation and *F* values of 4.74 km and 1.28, respectively, for a widespread stratiform rain event with embedded convection at the same location. In another study, Moreau et al. (2009) reported decorrelation and *F* values as 4.54 km and 1.3, respectively, based on a 1-year X-band polarimetric radar measurements at the Beauce region in Italy. It is to be noted, however, that using radars for rainfall statistical studies is biased by beam filtering effects (Jameson, 2017) so that any comparison with our method can only be taken qualitatively especially given the vast difference between the climate regimes. The gradient of correlation coefficient is significantly higher in case of Ahmedabad than Oklahoma due to change in rainfall characteristics. However, in spite of the obvious climatic differences, the characteristics of stratiform and convective rain are similar for both the locations.

To get an idea of the over all average characteristics of the spatial correlations of the study region, all available rain events were converted to the spatial rain structures using the proposed methodology. In Figure 3b variations of average values of correlation coefficients between rain rates at a pair of points with distance is plotted. The shaded portion indicates the standard deviation values. Here we use the same base of random number since the rain events are independent from each other and the variation of rain rate at each grid point for different events arises primarily due to the microphysical behavior of the original rain time series.

It can be noted that the correlation falls rapidly out to 10 km and then almost saturates or decreases slowly. However, the average correlation coefficient becomes half of the initial values within the first 2–3 km. The existence of such rapid fall indicates that there are significant number rain cells that are limited only to a few

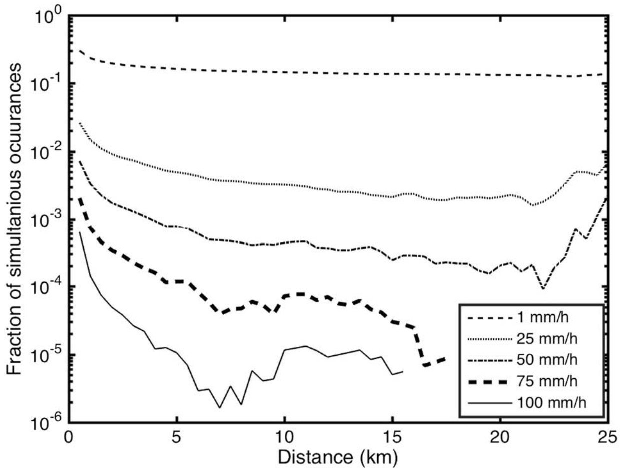

**Figure 4.** Spatial variations of occurrence probabilities of the same rain rate at two locations simultaneously.

kilometers (micro rain cell). The presence of three distinct gradient ranges (very fast, fast and slow) in the correlation coefficient with distance is indicative of presence of three types of rain cells having size <3, 3 – 10, and >10 km. A slight increase in correlation coefficient beyond 20 km is observed, and this may be due to the possibility of having another rain cell in that area (Yeo et al., 2011).

In Figure 4, the probability of simultaneous occurrences of rain rates greater than a fixed threshold are shown. High rain rates are less likely to occur at two locations simultaneously, a probability that decreases with the increasing separation. The maximum distance beyond which there is an increase in probability of simultaneous occurrences varies for different rain thresholds. In general, it is observed that this distance decreases with increase in rain rates. This occurs because high rain rate is associated with small rain cell sizes.

It can be concluded from Figures 3 and 4 that the rain rate follows an exponentially decreasing trend up to 20 km in Ahmedabad, and hence, diversity schemes can be modeled for a maximum distance of 20 km for this location. Beyond that, there is a possible decrease in diversity gains due to the presence of another rain cell. Accordingly, the site diversity analysis is restricted up to 20 km in the rest of the study.

### 3.3. Comparison of Diversity Gains of Different Regions

The comparative assessments of the proposed technique for site diversity study are carried out with two separate experimental studies where rain information is the primary source of data. The use of radar and networks of rain gauges are popular techniques for such purpose, and we compare our method with site diversity study of Yeo et al. (2011), which utilizes the radar data, and Matricciani (2003), which uses network of rain gauges. The results are also compared with ITU-R models.

In Figure 5, the variation of site diversity gain with distance at Ahmedabad estimated through proposed method is compared with the results reported from Italy by Matricciani (2003). To study the site diversity, Matricciani (2003) applied the synthetic storm technique (SST) to the rain time series measured by seven rain gauges along two linear arrays at Central (Fucino) and Northern Italy (Gera Lario). SST is used to convert the rain time series to the attenuation time series at 19.66 GHz along slant path elevation of 30.6° and 31.6°. The site diversity performance was then studied using the attenuation time series generated at varied distances up to 4.5 km.

In can be seen from Figure 5 that the proposed method at Ahmedabad and SST-derived results in Italy have similar diversity gains below 6-dB single-site attenuation value for distances above 2 km. For small distances, the diversity gain predicted at Ahmedabad is always lower than the measured values of Italy though the difference is small. For high single-site attenuation values, the observed diversity gain matches those predicted for large distances. Small rain attenuation usually occurs in stratiform types of rain with large rain cell sizes. This is the reason, then, why the predicted diversity gain at Ahmedabad matches well with that of the results obtained at Italy for low single-site attenuation values and large distances even though they are located in different climatic regions.

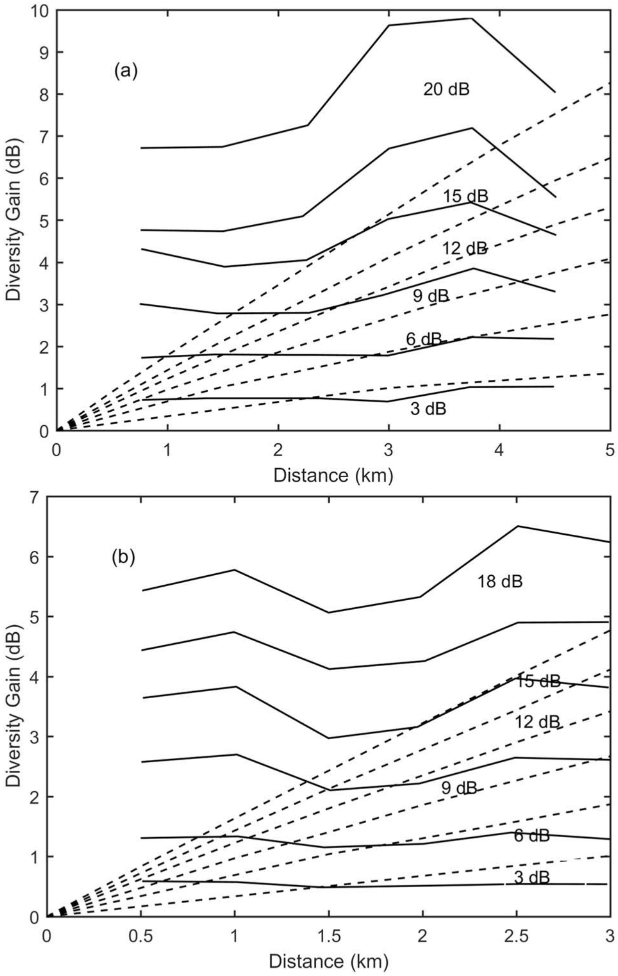

**Figure 5.** Performance comparison of the proposed method with measurements at two locations of Italy, (a) Fucino and (b) Gera Lario. Dotted lines indicate the simulation results of Ahmedabad and solid lines indicate the measured diversity gain in Italy. Single-site attenuation values are indicated along the associated curves. The diversity gains in Italy are estimated at 19.77 GHz with elevation angles 30.6° and 31.6° whereas in our model the same is estimated at 20 GHz with elevation angle of 30°. The Italian data are extracted from Matricciani (2003).

In Figure 6, the site diversity gain of Ahmedabad is plotted for a frequency of 20 GHz for single-site attenuation values along with the results from a similar study in Singapore (Yeo et al., 2011), which were based upon the radar data. The site separation distance is fixed at 15 km and elevation angle 50°. The original study for Singapore was made at a frequency of 18.9 GHz. However, they did not find any frequency dependence of the diversity gain in their study. Hence, we compare the diversity gain observed at Singapore directly with our result at 20 GHz. ITU-R (2015) model predicted site diversity gains are also plotted for the same set of parameters.

The diversity gain estimated using the proposed technique matches the Singapore data very well, but the simple ITU-R model shows significant underestimation. This is most likely due to the difference in the characteristics of the rain cells between tropical regions and those in a temperate region. The simple version of the ITU-R model is an adaptation of the Hodge model, which was developed for temperate regions. However, it must be remembered that high-intensity convective rain events are more frequent in tropical regions in contrast to the low-intensity widespread stratiform rain found frequently in temperate regions. Since the convective rain cells are much smaller in dimension than the stratiform rain cells, the diversity gain is much higher in tropical region than the temperate locations. The explicit version of the ITU-R model, however, matches well with the observed diversity gain for low single-site attenuation values. It overestimates the diversity gain for high single-site attenuation level above 8 dB. The explicit model of ITU-R is based on the joint probabilities of rain attenuation and is only useful when measurements are possible at two or more locations.

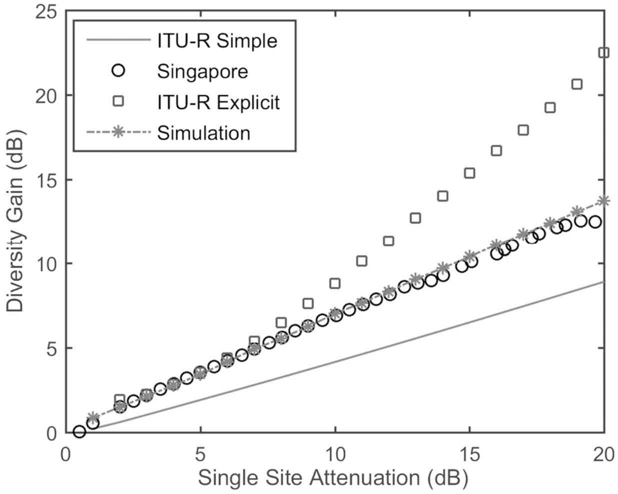

**Figure 6.** Performance comparison of the proposed method with ITU-R models and results from another tropical location of Singapore (distance = 15 km, elevation = 50°, baseline angle = 0°). The measurements at Singapore are at 18.9 GHz whereas ITU and our model is estimated at 20 GHz. The Singapore data are extracted from Yeo et al. (2011).

The estimation of joint probability is, even then, a challenging task with actual measurements. Here we utilize the typical values and code provided in ITU-R recommendations for estimation of diversity gain applicable for the Ahmedabad region. Further results are not compared with this version of the ITU-R model, and we limit our comparison with simple ITU-R model as these models utilize only single-site measurements.

As per the simple ITU-R model, the diversity gain depends on many parameters, the most important being frequency, site separation, elevation angle, and baseline angle. In the following subsections, the effect of different parameters on the site diversity performance is evaluated for Ahmedabad and compared with the simple ITU-R model.

### 3.4. Effect of Baseline Orientation

The baseline angle is the angle made by the line joining two earth stations with that of satellite path and varies between 0° and 90°. The ITU-R model incorporates the effect of baseline angle variation in diversity gain as a linear multiplicative factor. In Figure 7, the diversity gains estimated for Ahmedabad with baseline angles of 0° and 90° are compared with ITU-R model. It is observed that the effect of baseline angle on site diversity performance is insignificant, yet the ITU-R model shows an increase in diversity gain with baseline angle. Similar observations are also made by Yeo et al. (2011) using the radar data at Singapore. The reason for such behavior is explained by the wind direction and rain cell motion in their studies (Luini & Jeannin, 2009).

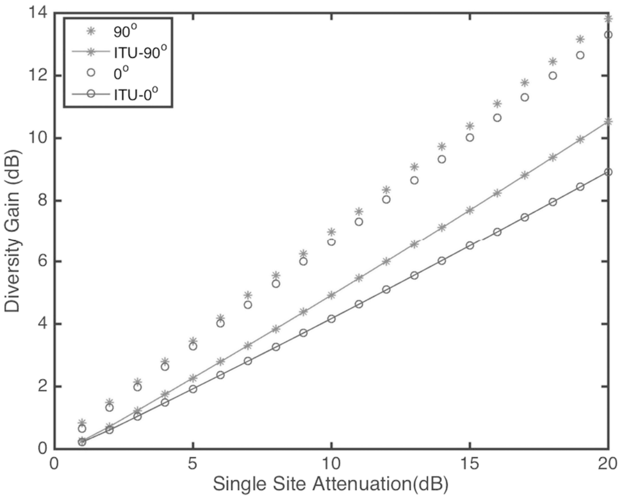

**Figure 7.** Variation of diversity gain with baseline orientation (distance = 15 km, elevation = 60°, frequency = 30 GHz).

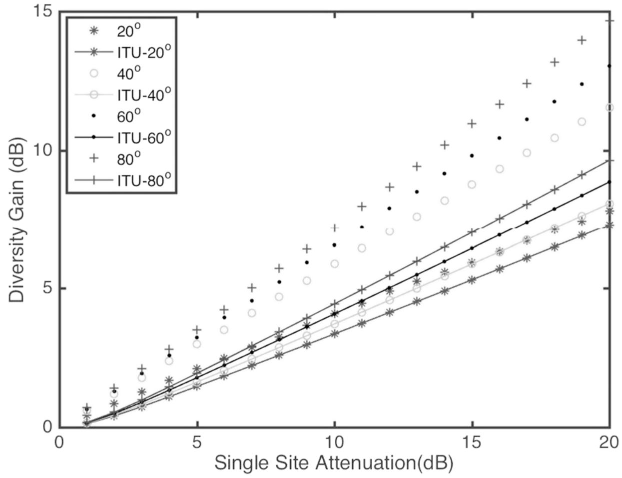

**Figure 8.** Variations of diversity gains with elevation angles (distance = 10 km, frequency = 30 GHz, baseline angle = 0°).

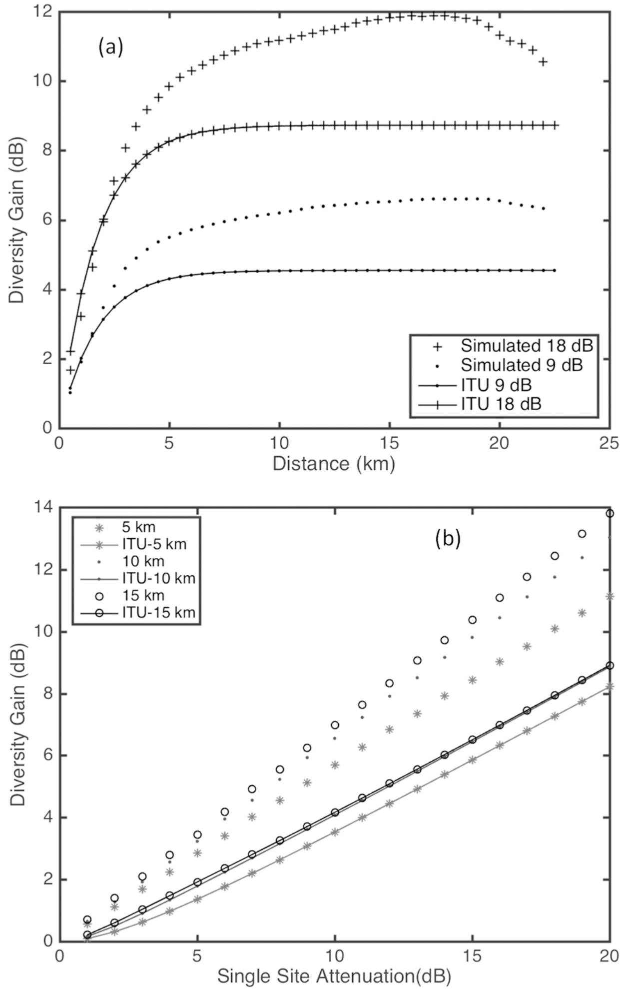

**Figure 9.** Variations of diversity gains with (a) distances for fixed attenuation values and (b) single-site attenuation values for fixed distances (elevation = 60°, frequency = 30 GHz, baseline angle = 0°).

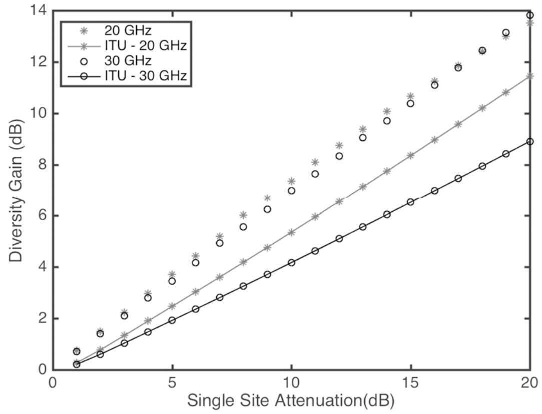

**Figure 10.** Variations of diversity gains with frequencies (distance = 15 km, elevation = 60°, baseline angle = 0°).

It is to be noted here that the diversity gain may vary for different baseline orientations during an isolated rain event because of azimuthal dependence of rain (Bringi et al., 2015; Yeo et al., 2011). Since in the present study wind motion is assumed to be same and in a fixed direction for all events, the effects of wind directions on site diversity cannot be quantified at this time. In future studies, however, this effect can be explored using the technique presented here. Nevertheless, this simple assumption may be quite representative since most of the rain occurs at Ahmedabad during June–September by the southwest monsoon.

### 3.5. Effect of Elevation Angle

For a fixed rain height, the elevation angle actually determines the length of satellite path that will be under the rain region. Hence, total attenuation suffered by the link will strongly depend upon the elevation angle. The effect of elevation angle is again assumed as a linear multiplicative factor in ITU-R model. In Figure 8, the effect of variations in elevation angle are shown along with those for the ITU-R model.

The simulation result in our study shows a similar trend to those observed for the ITU-R model. As the elevation angle increases, the path length through rain region decreases. Hence, the correlation between the amount of rain attenuation encountered by two locations far apart will also decrease.

Unlike the ITU-R model, the elevation angle of 20° shows significant low diversity gain as compared to the other elevation angles in the simulations. In very low elevation configuration, the path length become significantly large. This can lead to a condition where both links pass trough the same or multiple rain cells and hence decreasing the diversity gain. Similar observations are also reported by Yeo et al. (2011) where they found a drastic decrease in diversity gain below 30° elevation angle.

### 3.6. Effect of Site Separation Distance

Site separation distance has the most prominent effect on diversity gain. The rain rate is modeled as an exponentially decreasing function of distance in ITU-R model. In Figure 9a, the variation of diversity gain with distance is plotted for two single-site attenuation levels corresponding to fixed link availability percentages. In Figure 9b, the variation of diversity gain with single-site attenuation is plotted for varied distances.

It can be seen that ITU-R model matches with the our observed diversity gain only for low site separation distances (<3 km). Beyond 3 km, our observed diversity gain is higher than the ITU-R predicted values. The diversity gain saturates beyond 5 km in ITU-R model and similar trend is also observed for Ahmedabad. However, a little decrease in diversity gain can be noticed beyond 20 km in the simulation study. As already pointed out, the correlation between rain rates increases for large site separation distances and accordingly diversity gain decreases.

The difference between ITU-R and our observed values also increase with increase in single-site attenuation value. This is explored further in Figure 8b, which shows the variation of diversity gain with single-site attenuation for three different separation distances. The simulation result and ITU-R model both show a similar trend, but the magnitude of simulated diversity gain is always higher than the ITU-R model.

### 3.7. Effect of Frequency

The frequency plays an important role in determining the specific rain attenuation and, therefore, in the total rain attenuation. The frequency-dependent term in ITU-R model assumes a negative exponential form. This means that increases in frequency diversity gain decrease as a function of fixed site separations as shown in Figure 10. However, the simulation shows a much narrow difference between the diversity gains for different frequencies. In fact, the diversity gain for frequency 20 and 30 GHz is almost the same. Similar observations are also reported by Goldhirsh (1984) and Yeo et al. (2011), which also shows that the observed diversity gain is independent of frequency.

## 4. Discussions About the Assumptions Taken in the Simulation

The present method assumed the average wind speed in generating the spatial field. It is to be noted that the wind speed has an important role in determining the spatial resolution since the grid separation is estimated assuming storm motion uniform throughout the field, which is not always true. A more realistic field of nonuniform grid spacing can be generated from the instantaneous wind speed measurement. It can be argued that there will be all types of wind motions. However, when we consider all available pairs of points for the simulation, we still expect, the average diversity gain will be the same by using the average wind speed. This can be explored in future research.

It is also assumed that the direction of the wind speed is the same for all events, which is, again, not valid in a strict sense. However, as already pointed out, the majority of rain at Ahmedabad occurs due to southwest monsoon, in which the precipitation system is coming from a particular (southwest) direction. Hence, considering the wind direction might change the diversity gain for a single event when compared with different baseline angles but average statistics will remain the same (Bringi et al., 2015; Yeo et al., 2011). Moreover, such concerns can be addressed in later studies.

The finer spatial resolution can also be achieved in proposed method if we utilize high sampling rate of disdrometer. But the DSD measurements are then more susceptible to the error in small drop diameter range, which is minimized in case of large integration time as well as to the effects of statistical fluctuations.

## 5. Conclusions

Site diversity can ensure high link availability for low fade margin systems by utilizing spatial variability of rain field keeping the fade margin constant. However, the performance assessment of site diversity in tropical region is very limited due to the lack of simultaneous fade measurements from two or more satellite receiving stations and sparse radar coverages. Here we have proposed a copula-based Bayesian approach to predict the diversity performance from a single site's DSD measurement. The method is applied to the disdrometer measurements at Ahmedabad, a tropical location in India. The spatial correlation of rain estimated through this technique is compared with radar and disdrometer observations at Oklahoma. The diversity statistics is compared with ITU-R model and reported values from Singapore, another tropical location, and Italy, a temperate region. The results matches very well with the Singapore data. The diversity gain obtained at Italy also matches with Ahmedabad for low single-site attenuation values in spite of varied climate at these two locations. The observed diversity gain compares well with ITU-R model predicted values only for low single-site attenuation values or distances less than 3 km. For large distances or high single-site attenuation levels, the observed diversity gains differ significantly from the ITU-R predicted values indicating the different nature of rain cells in Ahmedabad. While these comparison results are encouraging further confirmation of the proposed method in different climatic conditions will be conducted in future studies as and when data will be made available.

Further investigations into the behavior of diversity schemes with different parameters such as distance, frequency, elevation angle, and baseline orientation reveal some interesting aspects of diversity schemes in tropical environment. These parameters are chosen since ITU-R model predicted diversity gain is a function of these parameters. It is found that rain rate are correlated up to 20 km in case of Ahmedabad, beyond which diversity gain decreases. It is further observed that frequency has practically no effect on diversity gain statistics for this location. It is also seen that the elevation angle dependence follows the same trend of ITU-R model, but, for very low elevation, diversity gain is significantly lower in comparison to the high elevation angles. From the study, it can be concluded that, in the absence of radar data, the proposed method can be a good and useful technique to study the site diversity performance utilizing only a single station's data.

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