

Joint statistics of rain rate and event duration for a tropical location in India

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Received 12 January 2009; revised 30 June 2009; re-revised received and accepted 27 October 2009

The optimum microwave link design requires the knowledge of rain rate distribution as well as of the duration statistics of the rain events. The rain rates measurements at Ahmedabad, a tropical location in India, are analysed and models for various event parameters are developed using three years of continuous measurements. The average number of cases with at least 1 min event duration shows an exponential dependence on the rainfall rate with a correlation coefficient of 0.93. Also, the average durations for different rainfall rates are found to follow a power law with a correlation coefficient of 0.97. It is found that the average duration more effectively represent the event duration at higher rain rates. The results are compared with those obtained for the temperate and other tropical locations to indicate the distinctiveness of the studied parameters over the location. It has been observed that although the tropical locations show similar qualitative features, the characteristics of the Indian region show quantitative difference from other tropical regions.

Keywords: Tropical rain, Rain rate, Event duration, Joint distribution

PACS No.: 92.40.eg

1 Introduction

The optimum utilisation of radio spectrum is a major concern of global radio communication community. In the microwave and millimeter region, rain is the significant cause of signal impairment in the form of fading¹⁻⁴. Strategies to combat the rain induced attenuation require a detailed understanding of the features of rainfall.

To mitigate the fade, it is required to know the exceeding probability of attenuation, which is mainly dependent on rainfall¹. But it is found that an assessment of link availability solely based on the rainfall exceeding statistics leads to over-designing of the microwave communication systems⁵. It is a well established fact that not only the intensity of rain, but also its duration is an important factor from the design point of view^{1,4-6}. System designers often need to decide whether to go for mitigation of fading or wait for the signal recovery. The rainfall occurrence probability alone is unable to answer the question. The distribution of event duration is also an important input in this aspect. In the temperate region, study has already been carried out to model the joint rainfall rate and duration statistics^{1,5,7}, but in the tropical region very few attempts have been made⁸ in this

respect although a few studies have been reported on rainfall characteristics from meteorological aspect⁹. In this paper, study on joint statistics of rain rate and duration of events for an Indian tropical location has been reported.

The Indian region has a great diversity in rain climatic conditions for different regions, especially for the Indian peninsula⁹. The results from a single location cannot represent this diversity but may provide some insight into features of tropical rain. The study emphasizes the variability of rain duration in temperate and tropical region and will be extended to variability among Indian locations in a future study when data will be available. However, the results obtained in this study are primarily applicable to Ahmedabad region and may be applicable to the locations in the same rain climatic zone.

The role of drop size distribution (DSD) in signal attenuation is important as the attenuation depends on the moment, between 3 and 4, of DSD. Also, DSD varies greatly with climatic regions^{10,11}. But measuring DSD is not a trivial issue for any location whereas rainfall intensity information is easier to get. The present study is focused on the practical applicability using simple method and readily

available data for designing satellite communication link at higher frequency bands.

In absence of actual fade measurement, detailed rain information is of particular importance. The conversion of the point rain rate distribution to the attenuation estimation, along horizontal or slant path, is far from trivial and needs reliable models¹². Few long term models, e.g. ITU-R model¹³ and Crane model¹⁴, based on rain rate information of many years are available. They can provide probabilistic value of rain attenuation exceeding a given percentage of time in a year. For generation of rain attenuation time series from rain rate time series, the synthetic storm technique (SST)^{15,16} is an effective method and is applicable to the links parallel to the rain advection. However, all these models are developed primarily with the temperate regions data and the extent of the applicability for Indian tropical region is not known due to the absence of actual measurements over the region. For this reason, duration statistics of rain is obtained in the present study rather than the simulated fade statistics. The fade duration can be estimated once the actual relation between fade and rain rate is established with GSAT-4 measurements.

The statistics of fade duration is relatively unknown for tropical region as models currently being used, have been developed based on temperate regions data^{17,18}. Existing ITU-R model P 530-12 (ref. 19) of fade duration on terrestrial link is based on the 1-year attenuation measurement at Scandinavian Peninsula at 18 GHz which derive coefficients a and b using power law relationship, $N_{10}(A)=aA^b$, between number of fade events (N_{10}) exceeding attenuation (A) for 10 s or longer. It is emphasized in the ITU-Recommendation that a and b depend on frequency, path length and climate. Similarly, ITU-R P 1623-1 (ref. 20) provides fade duration model applicable to satellite links assuming lognormal distribution of longer fades and power law relationship of shorter fades. Since the present study is done on a different climatic zone, this relation may not be applicable to this zone and needs further investigations. Results of this investigation on rain duration are presented in this paper and may further be validated with the availability of actual signal fade measurements.

The present study is particularly important as Indian Space Research Organization (ISRO) is currently planning to launch the Indian geostationary satellite GSAT-4 with a Ka-band beacon transmitter onboard to study the rain attenuation in Ka-band over

India²¹. The primary aim is to understand the rain attenuation characteristics for the development of suitable fade mitigation techniques (FMT). To support this propagation experiment, different meteorological parameters associated with rain are being measured at different locations in India. For the present study, rainfall measurements at Ahmedabad (23.06°N, 72.62°E), India have been utilized. The present work will provide a better understanding of the possible time extent of continuous rain outage to be encountered by the communication systems using readily available data such as rain rate. This will in turn be helpful to devise suitable FMT such as Time Diversity (TD)^{22,23} and Automatic Repeat reQuest (ARQ)²⁴ over this region.

2 Experimental techniques

At Ahmedabad (23.06°N, 72.62°E), located in the western part of India, rain mainly occurs in the monsoon period (i.e. July-September). However, the occurrence of rain at this location is low. The present study is based on the rainfall measurements for a period of three years (2005-2007). The data bank shows that the maximum rain rate observed at Ahmedabad was around 187 mm h⁻¹. At this location, rainfall measurements are made by a tipping bucket rain gauge and disdrometer.

The rain gauge, KWS032, indigenously developed by Space Application Centre, ISRO and manufactured by Komoline India Private Limited is according to the standard specified by India Meteorological Department (IMD). The instrument is operating since 2005. The instrument is a tipping bucket type and kept in an open ground around 150-200 m far from nearest building. It has collection area of 200 cm² and requires 0.25 ml of rain for a tip. The instrument records the all tipping times using a GPS receiver connected to the instrument. The instrument has a minimum time resolution of 1 tip s⁻¹.

The Joss-Waldvogel impact type disdrometer RD-80, manufactured by Distromet (Switzerland), is a rain drop size counter. It has a sensitive styrofoam cone connected with a transducer. When a drop strikes the cone, an electric signal is generated whose amplitude is proportional to the momentum of the drop. Using the Gunn-Kinzer relation²⁵, drop diameter is estimated from the terminal velocity. It is assumed that momentum is entirely due to the terminal velocity of the drops. It is also assumed that the drops are spherical in shape and no wind motion is present.

The catchment area of disdrometer is 50 cm² and it counts the number of drops in 127 size bins ranging drops of diameter 0.3 - 5 mm. The drops are regrouped by the instrument in 20 bins for statistical meaningful results. From the actual measurements of the drop size distribution, rain rate is given by

$$R = \frac{\pi}{6} * \frac{3.6}{10^3} * \frac{1}{A.t} \sum_{i=1}^{20} (n_i . D_i^3) \quad \dots(1)$$

where, A, is the total area of drop collection; t, integration time; n_i, number of drops for size class i; and D_i, mean diameter of class size i.

The sensitivity of disdrometer surface is very important for proper measurement²⁶. The surface is replaced once a year. The known sources of error like acoustic noises are minimized by proper installation of the instrument at the roof top of a building. Another error source of disdrometer is due to the insensitivity for a time period once a bigger drop strikes. This ‘dead time’ leads to under estimation of the smaller drops that fall within this period. But the effect of these smaller drops is less on rain intensity and is within 5% error limit²⁷. For this reason, in the present study, the dead time correction, which is prominent only for large drops, has not been applied.

The disdrometer and rain gauge are collocated with a separation of around 150-200 m. For this study, the rain rate obtained from the disdrometer has been considered. This is because small scale variation of rain rate has been considered, which is effectively averaged out in the rain gauge derived rain rate

variation. The acceptability of rain rate measurement from disdrometer measurement has also been tested and discussed.

3 Comparison of disdrometer and rain gauge measurements

The disdrometer counts the drop size and rain rate is obtained from the drop size distribution. On the other hand, the rain gauge measures the time period for the collection of a fixed amount of water. However, the tipping bucket rain gauge need large integration time particularly at low rain rates when the collection time is large and fast fluctuations in rain rates are time averaged. On the other hand, the disdrometer has a fixed integration time of 30 s in the present case.

A comparison between the rain gauge and disdrometer observations of rain rates during a rain event has been shown in Fig. 1 to check the capability of disdrometer in capturing the small scale temporal variation of rain rate. Rain rate of rain gauge is integrated over 30 s for comparison of data sets on similar scale as disdrometer has integration time of 30 s. Although this may include absolute error due to the linear downscaling of larger integration time measurement, their nature is expected to be similar. It is found that the two measurements are in good agreement although the rain gauge observations are not sensitive to fast fluctuations. In Fig. 2, the relation between the rain rates obtained from these two instruments has been shown. The linear curve fitted to the data has a gradient of 0.87, and the correlation

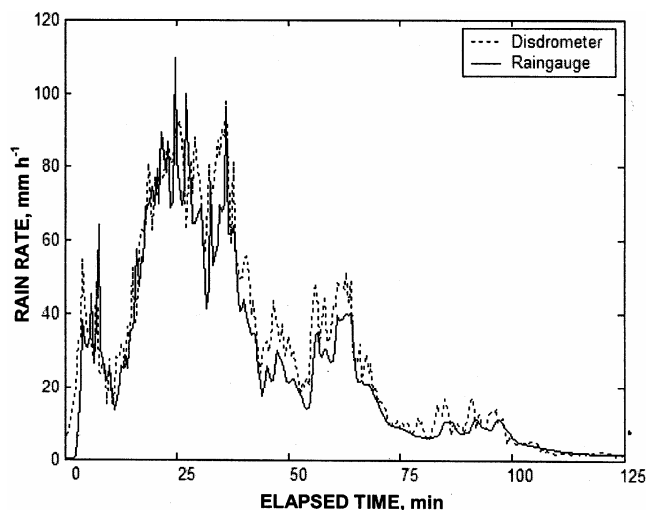


Fig. 1 — Rain rate variation obtained from disdrometer and rain gauge measurements

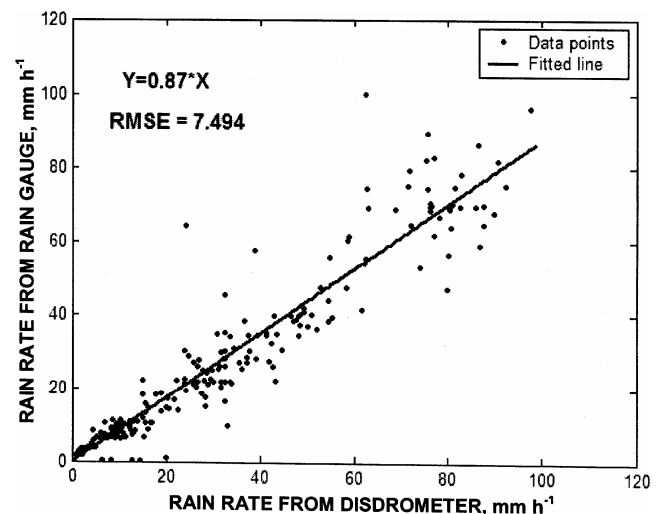


Fig. 2 — Relation between rain rates obtained from disdrometer and rain gauge

between two sets of data is 0.9 with root mean square error of 7.5 mm h^{-1} . This indicates that the disdrometer can generate good quality rain rate measurements with smaller integration time.

The rain accumulation between these two instruments, integrated over 24 h, is also shown in Fig. 3 along with the 1:1 relation line using all simultaneous measurement for the year 2007. The data is integrated over 24 h since rain gauge is more accurate over longer integration period. It shows that measurements from the two instruments follow a linear relationship, although disdrometer gives higher values than the rain gauge, especially at higher rain accumulation, and is in agreement with the previously reported study⁹.

Comparison of 3-year average rain rate exceeding probability of Ahmedabad with ITU-R model P 837-5 (ref. 28) is shown in Fig. 4. Average rain exceeding probability from rain gauge measurement for two years (2007-2008) is also shown in the same figure. It shows that rain rate, measured by both the instruments in the observation period, significantly differs with the ITU-R model. This shows that ITU-R model for this region is under estimated and may not be valid. It is observed that the disdrometer shows higher rain rate in comparison to the rain gauge below 0.1% exceeding probability. This is possibly due to higher sensitivity of the disdrometer compared to the rain gauge which is in agreement with the reported study⁹.

4 Statistical descriptions of rain events

A rain event is defined when continuous rain occurs above some predefined rain rate and time

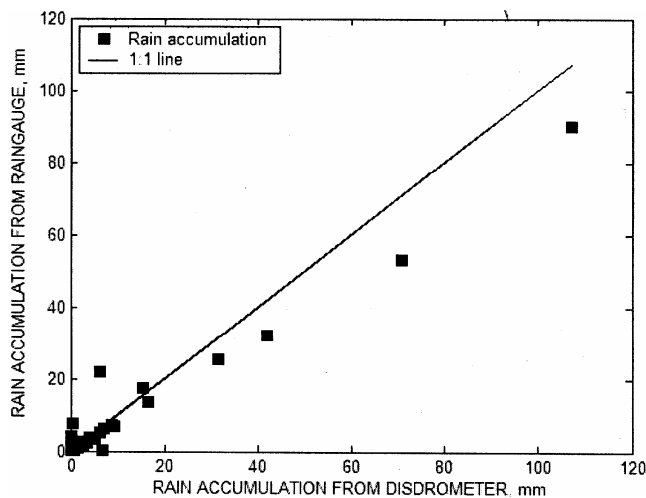


Fig. 3 — Relation between rain accumulations obtained from disdrometer and rain gauge for 2007. Each point corresponds to rain accumulation with integration time of 24 h

threshold^{19,20}. The idea of joint study of rain rate and duration is illustrated considering a typical rain event for Ahmedabad as shown in the Fig. 1. It is observed that the rain rate crosses the threshold of 20 mm h^{-1} four times and stayed above the rain rate with different time durations in each crossing. The 80 mm h^{-1} threshold of rain rate is also crossed four times but the time spans of these events are different. It is also clearly visible from Fig. 1 that the intermediate steps are also crossed with different frequency and durations. This is the general feature of all the rain events as they are inherently random and can therefore, be described only by statistical models.

The models of the event duration and the number of events are required for system design¹ and statistical distributions are generally required for this purpose^{1,6}. The average number of times a particular rain rate (R) is exceeded for longer than certain time duration (D) is important for knowing the rain occurrence pattern at a particular location. It is also to be noted that in general, average duration is biased towards the smaller duration region and does not necessarily represent the over all durations. However, it can be regarded as the initial input for a system design and can be extended to the real scenario by knowing the nature of duration distribution.

Different statistics of the rain event duration for Ahmedabad are modeled using average of all three years data. To understand the year-to-year variability of the results, error bar indicating the standard deviation of three years data have been shown. The results are compared with different tropical and

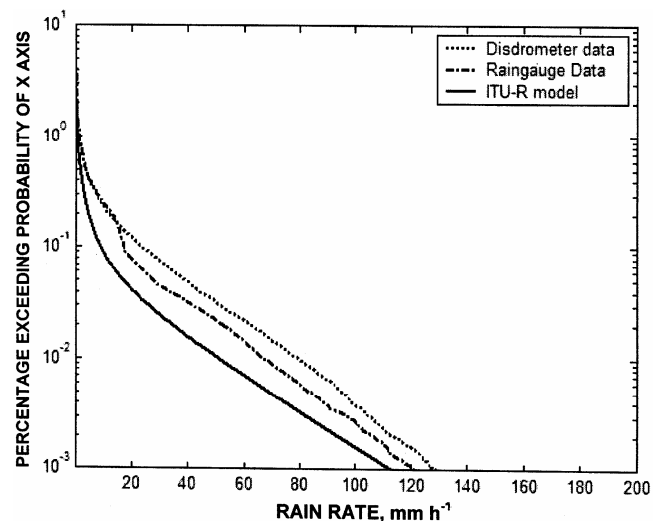


Fig. 4 — Comparison between measured and ITU-R rain rate exceeding probability of Ahmedabad

temperate locations. Although, the total observation period as well as the instrumental integration time are different for different regions, a comparison of averages for a year can qualitatively give an idea of the characteristic features. The comparisons are, however, with limited data from various locations.

5 Results and discussions

5.1 Statistics of rain rate and event duration

Figure 5 illustrates the average number of events (N) per year against the duration for different rain rates observed during three years (2005-2007). The total number of events above 10 mm h^{-1} with different durations for the total observation period of all the 3 years is 786. It is observed that as R increases N decreases. Also, for a particular value of R , N decreases with increase in D . This implies that lower rain rates ($10\text{-}30 \text{ mm h}^{-1}$) can continuously prevail over a longer duration, whereas, the higher rain rates are confined to a shorter time span in general. Although, this is the expected result, the similarity of curves for different rain rates implies that they follow a basic pattern whose parameters, however, depend on the rain climatology of the location. Thus, quantification of the event durations for different rain rates may provide useful insight to design communication systems.

It may be noted that rain rate curves representing greater than 140 mm h^{-1} are not shown in Fig. 5 as the rain rate above 140 mm h^{-1} is obtained on an average only once or twice a year with a duration of 30 s.

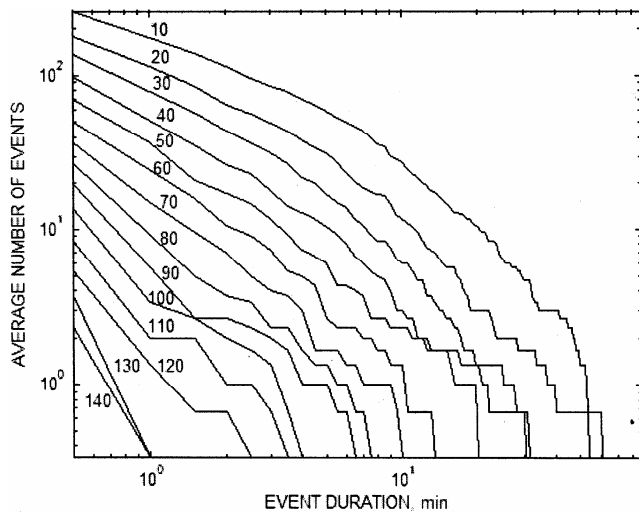


Fig. 5 — Contour plots giving average number of rain events against event duration at different rain rate thresholds. Numbers mentioned with contours give threshold rain rates

5.2 Empirical model of event number distribution

The average number of events of all durations for different rain rate has been computed and shown in Fig. 6. The distribution has been modeled in terms of the following expression:

$$\log_{10}(N) = -0.15 R + 2.58 \quad \dots(2)$$

Vilar *et al.*³ reported their model, also shown in Fig. 6, with the data from Barcelona, Spain (41.38°N , 2.15°E), a temperate location, as follows:

$$\log(N) = -8.97 \cdot 10^{-3} R + 2.01 \quad \dots(3)$$

It can be seen that the number of events at Ahmedabad are found to be more than that at Barcelona for rain rates up to 80 mm h^{-1} , but are comparable at higher rain rates.

The average number of events per year which last for at least 1 min for different rain rates has been computed and shown in Fig. 7. It is found that the distribution can be modeled as:

$$\log(N_{avg}) = -0.04 * R + 5.9 \quad \dots(4)$$

The correlation coefficient value for this fitting is found to be 0.93, indicating a very good fit between the observed and predicted values.

Table 1 gives a comparison of average number of events per year with at least 1 min duration at Ahmedabad (India), Slough (England)²⁹, Ile-Ife (Nigeria)⁶ and Barcelona (Spain)^{5,30}. It is found that Ahmedabad and Ile-Ife show comparable number of

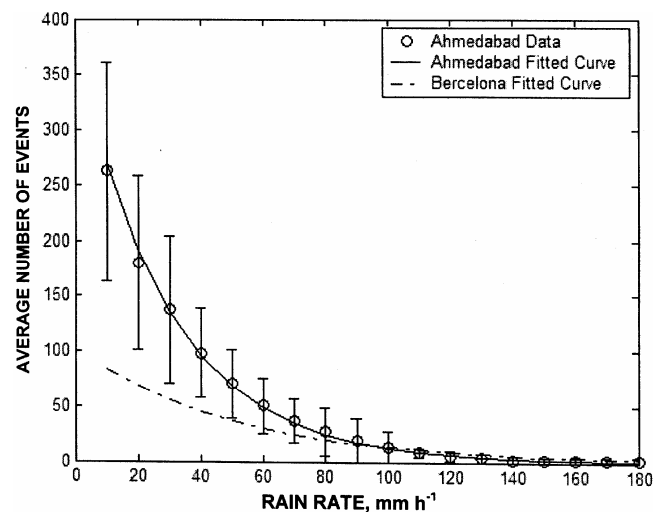


Fig. 6 — Average annual number of events in a year at Ahmedabad and Barcelona

events for rain rates up to 60 mm h⁻¹ whereas at Barcelona and Slough such events have much less occurrence. At Ahmedabad, the numbers of events are less than that occurred at other locations for rain rates above 100 mm h⁻¹, and the rain rate of ~80 mm h⁻¹ occurs for 0.01% of time at this location. Thus, numbers of events with at least 1 min duration at Ahmedabad may exceed that at temperate locations for 0.01% time in a year. This can further be an important parameter for link design with 99.99% service availability.

The numbers of events with 1 min duration at Barcelona at rain rates above 100 mm h⁻¹ are larger than that at Ahmedabad and this may be due to the fact that Barcelona is situated nearby sea coast. Also, the other tropical location Ile-Ife experienced more numbers of events for that rain rates at and above

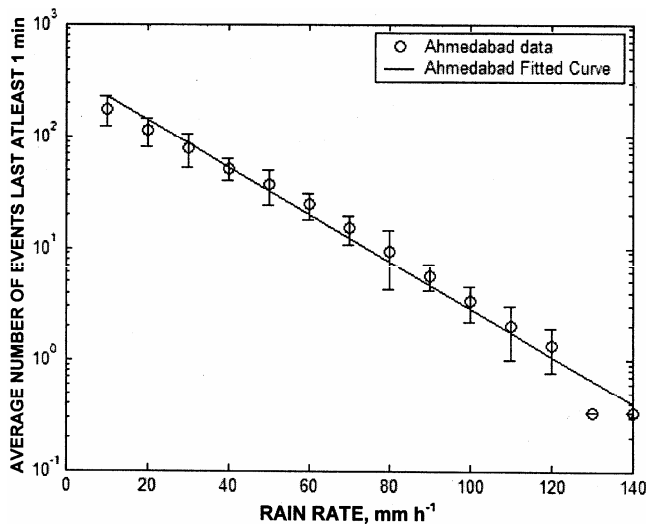


Fig. 7 — Average number of events which last at least 1 min for different rain rates at Ahmedabad

100 mm h⁻¹ indicating the rain characteristics at this location are different from that at Ahmedabad.

5.3 Empirical models of event duration distribution

The distributions of average durations (D_{avg}) of event against the rain rate for Ahmedabad and Barcelona have been shown in Fig. 8. The model for the duration for Ahmedabad can be obtained in terms of a power law as:

$$D_{avg} = 29.39 * R^{-0.74} \text{ min} \quad \dots(6)$$

The correlation coefficient value for the above fit is 0.97. The model for the data obtained at Barcelona is reported⁵ and is given by

$$D_{avg} = 12.23 * R^{-0.46} \text{ min} \quad \dots(7)$$

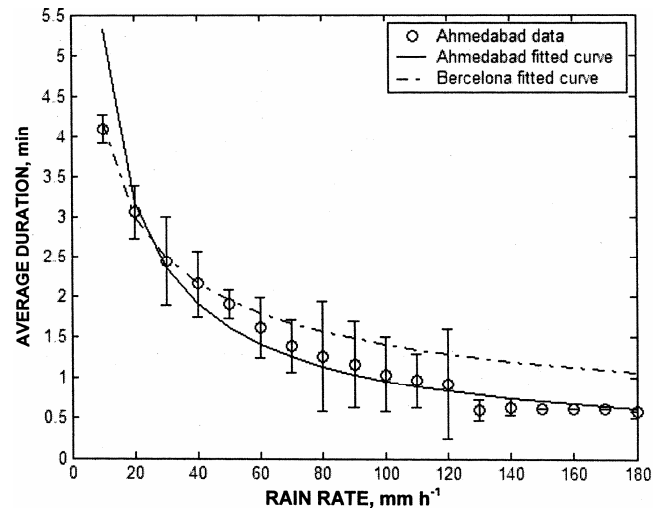


Fig. 8 — Average duration of events at Ahmedabad and Barcelona

Table 1 — Comparison of average number of events in a year with at least 1 min duration for a particular rain rate for different regions

R (mm h ⁻¹)	20	30	40	60	70	100	120	140	160	Integration period (s)	Total Observation time
Ahmedabad, India (23.06°N, 72.62°E)	113.3	79.0	51.3	24.3	15	3.3	1.3	0.33	-	30	36 months
Barcelona, Spain (41.38°N, 2.15°E)	60	40.4	21.8	16.8	12.9	6.2	3.9	2.4	1.5	20	49 yrs
Slough, England (51.5°N, 0.6°E)	17	6.5	3.5	-	1.8	-	-	-	-	-	5 yrs
Ile-Ife, Nigeria (7.5°N, 4.5°E)	100	-	-	40	-	9.5	4	1.3	0.4	10	26 months

‘-’ indicates absence of data

From the figure, it is observed that the average duration of rain rate is less at Ahmedabad than at Barcelona for rain rates above 30 mm h⁻¹.

It is to be noted that the average duration may strongly depend on the integration time of the instrument and measurement periods. However, the present results are helpful to provide an idea about the relative time duration of fades of different thresholds for this region.

6 Conclusions

The joint statistics of durations of rain events and rain rates are obtained and modelled for Ahmedabad. The relation between average duration and rain rate is found to follow the power law. The events associated with higher rain rates are found to be well characterized by the average duration distribution. The number of events is found to follow the exponential law with respect to rain rates.

The numbers of rain events with at least 1 min duration per year at different rain rates are compared for different locations in the tropical and temperate regions. At Ahmedabad, average duration is smaller for higher rain rates in comparison to Barcelona which is due to the fact that high rain rates corresponding to the short convective spell dominate in Ahmedabad region. Also, the number of cases with minimum 1 min duration with rain rate up to about 100 mm h⁻¹ at Ahmedabad is greater than that at the temperate locations. The information presented in this paper is expected to be of interest to system designers in setting up communication links in the microwave and millimeter wave bands in this part of the tropical region.

Acknowledgements

Authors gratefully acknowledge the facilities provided by Space Applications Centre, ISRO, Ahmedabad in gathering the experimental data used in the present study.

References

- Crane R K, *Electromagnetic wave propagation through rain* (John Wiley, New York), 1996.
- Ippolito J, *Radio Wave propagation in satellite communication* (Van Nostrand Reinhold, New York), 1986.
- Goldhirsh J, Rain-rate duration statistics over a five-year period: a tool for assessing slant path fade durations, *IEEE Trans Antennas Propag (USA)*, 43 (1995) 435.
- Paulson K S & Gibbins C J, Rain models for the prediction of fade durations at millimeter wavelengths, *IEE Proc Microw Antennas Propag (UK)*, 147 (2000) 431.
- Vilar E, Burgueno A, Puigcerver M & Austin J, Analysis of joint rainfall rate and duration statistics: Microwave system design implications, *IEEE Trans Commun (USA)*, 36 (1988) 650.
- Vilar E, Burgueno A & Ofoche E B C, Comparison of rainfall rate duration distributions for Ile-Ife and Barcelona, *Electron Lett (UK)*, 28 (1992) 1922.
- Vilar E & Burguefio A, Statistical properties of 49 years of rainfall rate events, *Theor Appl Climatol (Austria)*, 50 (1995) 213.
- Ajayi G O & Ofoche E B C, Some tropical rainfall rate characteristics at Ile-Ife for microwave and millimeter wave applications, *J Clim Appl Meteorol (USA)*, 23 (1974) 562.
- Sasi Kumar V, Sampath S, Vinayak P V S S K & Harikumar R, Rainfall intensity characteristics at coastal and high altitude stations in Kerala, *J Earth Sys Sci (India)*, 116 (2007) 451.
- Harikumar R, Sampath S & Sasi Kumar V, An empirical model for the variation of rain drop size distribution with rain rate at a few locations in Southern India, *Adv Space Res (UK)*, 43 (2009) 837.
- Harikumar R, Sasi Kumar V, Sampath S & Vinayak P V S S K, Comparison of drop size distribution between stations in eastern and western coasts of India, *J Indian Geophys Union*, 11 (2007) 116.
- Panagopoulos A D & Kanellopoulos J D, On the rain attenuation dynamics; spatial-temporal analysis of the rain fall rate and fade duration statistics, *Int J Satell Commun (UK)*, 21 (2003) 595
- Recommendation ITU-R - P 838-3: Specific attenuation model for rain for use in prediction methods* (International Telecommunication Union, Geneva), 2005.
- Crane R K, A two-component rain model for the prediction of attenuation statistics, *Radio Sci (USA)*, 17 (1982) 1371.
- Drufuca G, Rain attenuation statistics for frequencies above 10 GHz from rain gauge observations, *J Rech Atmos (France)*, 8 (1974) pp 399-411.
- Matricciani E, Physical-mathematical model of the dynamics of rain attenuation based on rain rate time series and a two-layer vertical structure of precipitation, *Radio Sci (USA)*, 31 (1996) 281.
- Cheffena M & Amaya C, Prediction model of fade duration statistics for satellite links between 10 and 50 GHz, *Antenna Wirel Propag Lett (USA)*, 7 (2008) 260.
- Paraboni A & Riva C, A new method for the prediction of fade duration statistics in satellite links above 10 GHz, *Int J Satell Commun (USA)*, 12 (1994) 387.
- Recommendation ITU-R - P 530-12: Propagation data and prediction methods required for the design of terrestrial line-of-sight systems* (International Telecommunication Union, Geneva), 2005.
- Recommendation ITU-R - P 1623-1: Prediction methods of fade dynamics on Earth-space path* (International Telecommunication Union, Geneva), 2005.
- Bandopadhyay K, GSAT-4, a step towards Indian advance communications satellite, *Int Workshop on Satellite and Space Communications* (Toulouse France), 2008.
- Ismail A F & Watson P A, Characteristics of fading and fade countermeasure on a satellite-earth link operating in an equatorial climate, with reference to broadcast application, *IEE Proc Microw Antennas Propag (UK)*, 147 (2000) 369.

- 23 Fukuchi H, Slant path attenuation analysis at 20 GHz for time-diversity reception of future satellite broadcasting, *URSI-F Open Symposium Colloque* (Ravenscar UK), 6.5.1 (1992)
- 24 Castanet L, Bolea-Alamanac A & Bousquet M, Interference and fade mitigation techniques for Ka and Q/V band satellite communication systems, *COST 280 (UK)*, 2005
- 25 Gunn R & Kinzer G D, The terminal velocity of fall for water droplets in stagnant air, *J Meteorol (USA)*, 8 (1949) 249.
- 26 Tokay A, Bashor P G & Wolff K R, Error characteristics of rainfall measurements by collocated Joss-Waldvogel disdrometers, *J Atmos Ocean Technol (USA)*, 22 (2005) 513.
- 27 Tokay, A & Short D, Evidence from tropical rain drop spectra of the origin of rain from stratiform versus convective, *J Appl Meteorol (USA)*, 35 (1996) 355.
- 28 *Reccomendation ITU-R-P.837-5 1: Characteristics of precipitation for propagation modeling* (International Telecommunication Union, Geneva), 2007.
- 29 Harden B N, Norbury J R & White W J K, Measurements of rainfall for studies of millimeter radio attenuation, *IEE Proc Microw Opt Acous (UK)*, 16 (1977) 197.
- 30 Paulson K S, High temporal resolution rain duration statistics in the southern UK, *IEE Proc Microw Antennas Propag (UK)*, 151 (2004) 375.